

D-wave charmed and bottomed baryons from QCD sum rules

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We study the D -wave charmed baryons of $SU(3)$ flavor $\bar{\mathbf{3}}_F$ using the method of QCD sum rules in the framework of heavy quark effective theory. We find that the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$ can be well described by the D -wave $SU(3)$ $\bar{\mathbf{3}}_F$ charmed baryon multiplets of $J^P = 3/2^+$ and $5/2^+$, which contain two λ -mode orbital excitations, i.e., the $\Lambda_c(2880)$ has $J^P = 5/2^+$, and the $\Xi_c(3055)$ and $\Xi_c(3080)$ have $J^P = 3/2^+$ and $5/2^+$, respectively. Our results also suggest that the $\Lambda_c(2880)$ has a partner state, the $\Lambda_c(3/2^+)$ of $J^P = 3/2^+$. Its mass is around $2.81^{+0.33}_{-0.18}$ GeV, and the mass difference between it and the $\Lambda_c(2880)$ is 28^{+45}_{-24} MeV. We also evaluate the masses of their bottom partners.

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I. INTRODUCTION

In the past years important experimental progresses have been made in the field of charmed baryons. All the $1S$ charmed baryons have been well established [1]. Moreover, the $1P$ states $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Xi_c(2790)$ and $\Xi_c(2815)$ have also been well observed and complete two $SU(3)$ flavor $\bar{\mathbf{3}}_F$ multiplets of $J^P = 1/2^-$ and $3/2^-$ [2–5]. Besides them, there still exist many higher states, i.e., the $\Lambda_c(2765)$ ($J^P = ?^?$) [6], the $\Lambda_c(2880)$ ($J^P = 5/2^+$) [6], the $\Lambda_c(2940)$ ($J^P = ?^?$) [7, 8], the $\Sigma_c(2800)$ ($J^P = ?^?$) [9], the $\Xi_c(2930)$ ($J^P = ?^?$) [10], the $\Xi_c(2980)$ ($J^P = ?^?$) [11, 12], the $\Xi_c(3055)$ ($J^P = ?^?$) [13, 14], the $\Xi_c(3080)$ ($J^P = ?^?$) [11], and the $\Xi_c(3123)$ ($J^P = ?^?$) [13]. Some of them may belong to the $1P$ $SU(3)$ flavor $\mathbf{6}_F$ multiplets, while some of them are good D -wave charmed baryon candidates. Especially, in this paper we shall concentrate on the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$, which were proposed (or detailly discussed) in Refs. [15–17] to be $1D$ charmed baryons of the quantum numbers $J^P = 5/2^+$, $3/2^+$ and $5/2^+$, respectively. More assignments can be found in Refs. [18–22], and we refer to reviews [17, 23] for their recent progress.

The charmed baryons have been investigated using many phenomenological methods/models in the past two decades, including various quark models [24–28], the combined expansion in $1/m_Q$ and $1/N_c$ [29], the hyperfine interaction [30, 31], the Feynman-Hellmann theorem [32], the variational approach [33], the unitarized dynamical model [34], the extended local hidden gauge approach [35], the unitarized chiral perturbation theory [36], and the Lattice QCD [37–39], etc. Their pionic decays and related pion induced reactions have also been studied in Refs. [21, 40, 41]. See reviews in Refs. [42–45].

We have also systematically studied the charmed baryons, i.e., the S -wave bottom baryons [46], the P -wave charmed baryons [47], and the P -wave bottom baryons [48], using the method of QCD sum rules [49, 50] in the framework of heavy quark effective theory (HQET) [51–53]. This scheme has been successfully applied to study heavy mesons and baryons containing a single heavy quark [54–76], while other studies using the method of QCD sum rules but not in HQET can be found in Refs. [77–82].

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In this paper we study the D -wave charmed baryons of $SU(3)$ flavor $\bar{\mathbf{3}}_F$ (Λ_c, Ξ_c) using the method of QCD sum rules within HQET. This paper is organized as follows. First we systematically construct the interpolating currents for the D -wave charmed baryons in Sec. II. Then we select some of them to perform the QCD sum rule analysis at both the leading order in Sec. III and the order $\mathcal{O}(1/m_Q)$ in Sec. IV. During the calculations, we shall take the $\mathcal{O}(1/m_c)$ corrections (m_c is the heavy quark mass) into account, and extract the chromomagnetic splitting. In Sec. V we perform numerical analyses and discuss the obtained results. A short summary is given in Sec. VI.

II. INTERPOLATING FIELDS FOR THE P -WAVE CHARMED BARYON

The charmed baryons of P - and D -waves have been systemically classified in Ref. [83], where their strong decays were systematically investigated using the 3P_0 model. The P -wave charmed baryon interpolating fields have been systematically constructed in Refs. [47, 48] using the same notations, i.e., l_ρ denotes the orbital angular momentum between the two light quarks and l_λ denotes the orbital angular momentum between the charm quark and the two-light-quark system.

In this paper we follow the same approach of Refs. [47, 48], and construct the D -wave ($L = 2$) charmed baryon interpolating fields. We use the notation $J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2}}$ to denote the D -wave charmed baryon interpolating field having the total angular momentum j and parity P , and belonging to the spin doublet $[F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda]$. Here F denotes the $SU(3)$ flavor representation, either $\bar{\mathbf{3}}_F$ or $\mathbf{6}_F$; j_l and s_l denote the total angular momentum and spin angular momentum of the light components; $[\rho\rho]$ denotes $l_\rho = 2$ and $l_\lambda = 0$, $[\lambda\lambda]$ denotes $l_\rho = 0$ and $l_\lambda = 2$, and $[\rho\lambda]$ denotes $l_\rho = 1$ and $l_\lambda = 1$. We have the relations $L = l_\lambda \otimes l_\rho$, $j_l = L \otimes s_l$ and $j = j_l \otimes s_Q$, where $s_Q = 1/2$ is the spin of the heavy quark.

We summarize all the possible configurations of the D -wave ($L = 2$) charmed baryons in Fig. 1, where **A** and **S** denote the structure to be antisymmetric and symmetric, respectively. We note that the type $[\rho\lambda]$ ($l_\rho = 1$ and $l_\lambda = 1$) can actually have total orbital angular momenta $L = 0, 1$ and 2 , but in this paper we only concentrate on the $L = 2$ for the D -wave case.

Generally, the interpolating field for charmed baryons can be written as a combination of a diquark field and a heavy quark field

$$J(x) \sim \epsilon_{abc} (q^{aT}(x) \mathbb{C} \Gamma_1 q^b(x)) \Gamma_2 h_v^c(x), \quad (1)$$

where a, b and c are color indices; ϵ_{abc} is the totally antisymmetric tensor; the superscript T represents the transpose of the Dirac indices; the matrices $\Gamma_{1,2}$ are Dirac matrices which describe the Lorentz structure; \mathbb{C} is the charge-conjugation operator; $q(x)$ denotes the light quark field at location x , and it can be either $u(x)$ or $d(x)$ or $s(x)$; $h_v(x)$ denotes the heavy quark field, and we have used the Fierz transformation to move it to the rightmost place. Besides these notations, $\gamma_\mu^t = \gamma_\mu - \not{v}v_\mu$, $D_\mu = \partial_\mu - igA_\mu$, $D_\mu^t = D_\mu - (D \cdot v)v_\mu$, v is the velocity of the heavy quark, and $g_t^{\alpha_1\alpha_2} = g^{\alpha_1\alpha_2} - v^{\alpha_1}v^{\alpha_2}$ is the transverse metric tensor.

To describe the orbital angular momenta, we directly apply two derivatives containing two symmetric Lorentz indices on the light diquark field (see Refs. [47, 48, 73–76] for more details) to construct the D -wave diquark fields of

$$\begin{aligned}
& [\rho\rho] \quad l_\rho = 2 \text{ (S)}, \quad l_\lambda = 0, \quad \bar{\mathbf{3}}_C \text{ (A)} \\
& \quad \bar{\mathbf{3}}_F \text{ (A): } L = 2 \otimes s_l = 0 \text{ (A)} \longrightarrow j_l = 2: \Lambda_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad (\rho\rho\text{-a}) [\bar{\mathbf{3}}_F, 2, 0, \rho\rho] \\
& \quad \mathbf{6}_F \text{ (S): } L = 2 \otimes s_l = 1 \text{ (S)} \begin{cases} j_l = 1: \Sigma_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) \quad \Xi_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) \quad \Omega_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) & (\rho\rho\text{-b}) [\mathbf{6}_F, 1, 1, \rho\rho] \\ j_l = 2: \Sigma_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Omega_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) & (\rho\rho\text{-c}) [\mathbf{6}_F, 2, 1, \rho\rho] \\ j_l = 3: \Sigma_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) \quad \Xi_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) \quad \Omega_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) & (\rho\rho\text{-d}) [\mathbf{6}_F, 3, 1, \rho\rho] \end{cases} \\
& [\lambda\lambda] \quad l_\rho = 0 \text{ (S)}, \quad l_\lambda = 2, \quad \bar{\mathbf{3}}_C \text{ (A)} \\
& \quad \bar{\mathbf{3}}_F \text{ (A): } L = 2 \otimes s_l = 0 \text{ (A)} \longrightarrow j_l = 2: \Lambda_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad (\lambda\lambda\text{-a}) [\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda] \\
& \quad \mathbf{6}_F \text{ (S): } L = 2 \otimes s_l = 1 \text{ (S)} \begin{cases} j_l = 1: \Sigma_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) \quad \Xi_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) \quad \Omega_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) & (\lambda\lambda\text{-b}) [\mathbf{6}_F, 1, 1, \lambda\lambda] \\ j_l = 2: \Sigma_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Omega_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) & (\lambda\lambda\text{-c}) [\mathbf{6}_F, 2, 1, \lambda\lambda] \\ j_l = 3: \Sigma_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) \quad \Xi_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) \quad \Omega_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) & (\lambda\lambda\text{-d}) [\mathbf{6}_F, 3, 1, \lambda\lambda] \end{cases} \\
& [\rho\lambda] \quad l_\rho = 1 \text{ (A)}, \quad l_\lambda = 1, \quad \bar{\mathbf{3}}_C \text{ (A)} \\
& \quad \mathbf{6}_F \text{ (S): } L = 2 \otimes s_l = 0 \text{ (A)} \longrightarrow j_l = 2: \Sigma_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Omega_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad (\rho\lambda\text{-a}) [\mathbf{6}_F, 2, 0, \rho\lambda] \\
& \quad \bar{\mathbf{3}}_F \text{ (A): } L = 2 \otimes s_l = 1 \text{ (S)} \begin{cases} j_l = 1: \Lambda_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) \quad \Xi_{c1} \left(\frac{1}{2}^+, \frac{3}{2}^+ \right) & (\rho\lambda\text{-b}) [\bar{\mathbf{3}}_F, 1, 1, \rho\lambda] \\ j_l = 2: \Lambda_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) \quad \Xi_{c2} \left(\frac{3}{2}^+, \frac{5}{2}^+ \right) & (\rho\lambda\text{-c}) [\bar{\mathbf{3}}_F, 2, 1, \rho\lambda] \\ j_l = 3: \Lambda_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) \quad \Xi_{c3} \left(\frac{5}{2}^+, \frac{7}{2}^+ \right) & (\rho\lambda\text{-d}) [\bar{\mathbf{3}}_F, 3, 1, \rho\lambda] \end{cases} \\
& \quad L = 0/1 : \text{ we do not study these two cases in this paper.}
\end{aligned}$$

FIG. 1: The notations for D -wave charmed baryons: $\mathbf{6}_F$ (S) and $\bar{\mathbf{3}}_F$ (A) denote the $SU(3)$ flavor representations; $\bar{\mathbf{3}}_C$ (A) denotes the $SU(3)$ color representation; s_l is the spin angular momentum of the two light quarks; $j_l = L \otimes s_l = l_\lambda \otimes l_\rho \otimes s_l$ is the total angular momentum of the two light quarks.

the configuration $[\rho\rho/\lambda\lambda/\rho\lambda]$:

$$\begin{aligned}
& [\rho\rho] \quad [^1D_2] : \quad l_\rho = 2 \text{ (S)}, \quad l_\lambda = 0, \quad L = 2, \quad s_l = 0 \text{ (A)}, \quad j_l = 2 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - 2[\mathcal{D}_{\mu_1} q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2} q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\rho\rho] \quad [^3D_{1/2/3}] : \quad l_\rho = 2 \text{ (S)}, \quad l_\lambda = 0, \quad L = 2, \quad s_l = 1 \text{ (A)}, \quad j_l = 1/2/3 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_\mu q^b(x) - 2[\mathcal{D}_{\mu_1} q^{aT}(x)] \mathbb{C} \gamma_\mu [\mathcal{D}_{\mu_2} q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_\mu [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\lambda\lambda] \quad [^1S_0] : \quad l_\rho = 0 \text{ (S)}, \quad l_\lambda = 2, \quad L = 2, \quad s_l = 0 \text{ (A)}, \quad j_l = 2 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) + 2[\mathcal{D}_{\mu_1} q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2} q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\lambda\lambda] \quad [^3S_1] : \quad l_\rho = 0 \text{ (S)}, \quad l_\lambda = 2, \quad L = 2, \quad s_l = 1 \text{ (A)}, \quad j_l = 1/2/3 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_\mu q^b(x) + 2[\mathcal{D}_{\mu_1} q^{aT}(x)] \mathbb{C} \gamma_\mu [\mathcal{D}_{\mu_2} q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_\mu [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\rho\lambda] \quad [^1P_1] : \quad l_\rho = 1 \text{ (A)}, \quad l_\lambda = 1, \quad L = 2, \quad s_l = 0 \text{ (A)}, \quad j_l = 2 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\rho\lambda] \quad [^3P_{0/1/2}] : \quad l_\rho = 1 \text{ (A)}, \quad l_\lambda = 1, \quad L = 2, \quad s_l = 1 \text{ (A)}, \quad j_l = 1/2/3 \\
& \quad \epsilon_{abc} \times \left([\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^{aT}(x)] \mathbb{C} \gamma_\mu q^b(x) - q^{aT}(x) \mathbb{C} \gamma_\mu [\mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} q^b(x)] \right) + \mu_1 \leftrightarrow \mu_2, \\
& [\rho\lambda] \quad [\cdots] : \quad l_\rho = 1 \text{ (A)}, \quad l_\lambda = 1, \quad L = 0/1, \quad s_l = 0/1 \text{ (A/S)}, \quad j_l = 0/1/2 \\
& \quad \text{we do not study these cases in this paper.}
\end{aligned}$$

In these expressions, we have used $[^{2s_l+1}(l_\rho)_{l_\rho \otimes s_l}]$ to denote the spin, orbital and total angular momenta of the diquark, where l_λ (the orbital angular momentum between the charm quark and the diquark) is not taken into account. Especially, $[^3D_{1/2/3}]$ means $l_\rho \otimes s_l$ can be 1, 2 and 3, while $[^3P_{0/1/2}]$ means $l_\rho \otimes s_l$ can be 0, 1 and 2.

Based on these D -wave diquark fields, we can construct the D -wave ($L = 2$) charmed baryons of the configuration $[\rho\rho/\lambda\lambda/\rho\lambda]$:

- $[\rho\rho]$ ($l_\rho = 2$ (**S**) and $l_\lambda = 0$):

($\rho\rho$ -a) $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$ with $s_l = 0$ (**A**) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (**A**) and flavor $\bar{\mathbf{3}}_F$ (**A**), and we obtain a spin doublet ($j^P = 3/2^+, 5/2^+$):

$$J_{3/2,+, \bar{\mathbf{3}}_F, 2, 0, \rho\rho}^\alpha(x) \quad (2)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \left(\frac{1}{2} g_t^{\mu_1 \alpha} g_t^{\mu_2 \mu_4} + \frac{1}{2} g_t^{\mu_2 \alpha} g_t^{\mu_1 \mu_4} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_4 \alpha} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{5/2,+, \bar{\mathbf{3}}_F, 2, 0, \rho\rho}^{\alpha_1 \alpha_2}(x) \quad (3)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \Gamma_t^{\alpha_1 \alpha_2, \mu_1 \mu_2} \times h_v^c(x),$$

where $\Gamma_t^{\alpha_1 \alpha_2, \mu_1 \mu_2}$ is the projection operator projecting into pure spin 2, whose explicit form is given in Appendix A.

($\rho\rho$ -b) $[\mathbf{6}_F, 1, 1, \rho\rho]$ with $s_l = 1$ (**S**) and $j_l = 1$. Now the diquark has color $\bar{\mathbf{3}}_C$ (**A**) and flavor $\mathbf{6}_F$ (**S**), and we obtain a spin doublet ($1/2^+, 3/2^+$):

$$J_{1/2,+, \mathbf{6}_F, 1, 1, \rho\rho}(x) \quad (4)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \left(g_t^{\mu_1 \mu_3} g_t^{\mu_2 \mu_4} + g_t^{\mu_2 \mu_3} g_t^{\mu_1 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{3/2,+, \mathbf{6}_F, 1, 1, \rho\rho}^\alpha(x) \quad (5)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \left(\frac{1}{2} g_t^{\mu_1 \mu_3} g_t^{\mu_2 \alpha} + \frac{1}{2} g_t^{\mu_2 \mu_3} g_t^{\mu_1 \alpha} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_3 \alpha} \right) \times h_v^c(x).$$

($\rho\rho$ -c) $[\mathbf{6}_F, 2, 1, \rho\rho]$ with $s_l = 1$ (**S**) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (**A**) and flavor $\mathbf{6}_F$ (**S**), and we obtain a spin doublet ($3/2^+, 5/2^+$). We failed to construct these currents because we do not know how to explicitly combine angular momenta $J = 2$ and $J = 1$ to be $J = 2$, i.e., how to use two symmetric indices $\{\mu_1 \mu_2 + \mu_2 \mu_1\}$ and another index μ_3 to obtain two symmetric indices $\{\alpha_1 \alpha_2 + \alpha_2 \alpha_1\}$. To estimate the masses of these states, we shall use the currents of ($\rho\rho$ -b) and ($\rho\rho$ -d) as explained in Sec. VI.

($\rho\rho$ -d) $[\mathbf{6}_F, 3, 1, \rho\rho]$ with $s_l = 1$ (**S**) and $j_l = 3$. Now the diquark has color $\bar{\mathbf{3}}_C$ (**A**) and flavor $\mathbf{6}_F$ (**S**), and we obtain a spin doublet ($5/2^+, 7/2^+$):

$$J_{5/2,+, \mathbf{6}_F, 3, 1, \rho\rho}^{\alpha_1 \alpha_2}(x) \quad (6)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \Gamma_t^{\alpha_1 \alpha_2, \nu_1 \nu_2} \times \left(g_t^{\mu_1 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_3 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_1 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_1 \nu_2} g_t^{\mu_2 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{7/2,+, \mathbf{6}_F, 3, 1, \rho\rho}^{\alpha_1 \alpha_2 \alpha_3}(x) \quad (7)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \Gamma_t^{\alpha_1 \alpha_2 \alpha_3, \mu_1 \mu_2 \mu_3} \times h_v^c(x),$$

where $\Gamma_t^{\alpha_1 \alpha_2 \alpha_3, \mu_1 \mu_2 \mu_3}$ is the projection operator projecting into pure spin 3.

- $[\lambda\lambda]$ ($l_\rho = 0$ (**S**) and $l_\lambda = 2$):

($\lambda\lambda$ -a) $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$ with $s_l = 0$ (\mathbf{A}) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\bar{\mathbf{3}}_F$ (\mathbf{A}), and we obtain a spin doublet ($j^P = 3/2^+, 5/2^+$):

$$\begin{aligned} & J_{3/2,+, \bar{\mathbf{3}}_F, 2, 0, \lambda\lambda}^\alpha(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \left(\frac{1}{2} g_t^{\mu_1 \alpha} g_t^{\mu_2 \mu_4} + \frac{1}{2} g_t^{\mu_2 \alpha} g_t^{\mu_1 \mu_4} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_4 \alpha} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x), \end{aligned} \quad (8)$$

$$\begin{aligned} & J_{5/2,+, \bar{\mathbf{3}}_F, 2, 0, \lambda\lambda}^{\alpha_1 \alpha_2}(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \Gamma_t^{\alpha_1 \alpha_2, \mu_1 \mu_2} \times h_v^c(x). \end{aligned} \quad (9)$$

($\lambda\lambda$ -b) $[\mathbf{6}_F, 1, 1, \lambda\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 1$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\mathbf{6}_F$ (\mathbf{S}), and we obtain a spin doublet ($1/2^+, 3/2^+$):

$$\begin{aligned} & J_{1/2,+, \mathbf{6}_F, 1, 1, \lambda\lambda}(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \left(g_t^{\mu_1 \mu_3} g_t^{\mu_2 \mu_4} + g_t^{\mu_2 \mu_3} g_t^{\mu_1 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x), \end{aligned} \quad (10)$$

$$\begin{aligned} & J_{3/2,+, \mathbf{6}_F, 1, 1, \lambda\lambda}(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \left(\frac{1}{2} g_t^{\mu_1 \mu_3} g_t^{\mu_2 \alpha} + \frac{1}{2} g_t^{\mu_2 \mu_3} g_t^{\mu_1 \alpha} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_3 \alpha} \right) \times h_v^c(x). \end{aligned} \quad (11)$$

($\lambda\lambda$ -c) $[\mathbf{6}_F, 2, 1, \lambda\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\mathbf{6}_F$ (\mathbf{S}), and we obtain a spin doublet ($3/2^+, 5/2^+$). We failed to construct these currents.

($\lambda\lambda$ -d) $[\mathbf{6}_F, 3, 1, \lambda\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 3$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\mathbf{6}_F$ (\mathbf{S}), and we obtain a spin doublet ($5/2^+, 7/2^+$):

$$\begin{aligned} & J_{5/2,+, \mathbf{6}_F, 3, 1, \lambda\lambda}^{\alpha_1 \alpha_2}(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \Gamma_t^{\alpha_1 \alpha_2, \nu_1 \nu_2} \times \left(g_t^{\mu_1 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_3 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_1 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_1 \nu_2} g_t^{\mu_2 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x), \\ & J_{7/2,+, \mathbf{6}_F, 3, 1, \lambda\lambda}^{\alpha_1 \alpha_2 \alpha_3}(x) \\ &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) + 2[\mathcal{D}_{\mu_1}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_2}^t q^b(x)] + q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \Gamma_t^{\alpha_1 \alpha_2 \alpha_3, \mu_1 \mu_2 \mu_3} \times h_v^c(x). \end{aligned} \quad (12)$$

• $[\rho\lambda]$ ($l_\rho = 1$ (\mathbf{A}) and $l_\lambda = 1$):

($\rho\lambda$ -a) $[\mathbf{6}_F, 2, 0, \rho\lambda]$ with $s_l = 0$ (\mathbf{A}) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\mathbf{6}_F$ (\mathbf{S}), and we obtain a spin doublet ($j^P = 3/2^+, 5/2^+$):

$$\begin{aligned} J_{3/2,+, \mathbf{6}_F, 2, 0, \rho\lambda}^\alpha(x) &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \left(\frac{1}{2} g_t^{\mu_1 \alpha} g_t^{\mu_2 \mu_4} + \frac{1}{2} g_t^{\mu_2 \alpha} g_t^{\mu_1 \mu_4} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_4 \alpha} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x), \end{aligned} \quad (14)$$

$$\begin{aligned} J_{5/2,+, \mathbf{6}_F, 2, 0, \rho\lambda}^{\alpha_1 \alpha_2}(x) &= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_5 q^b(x) - q^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \\ & \quad \times \Gamma_t^{\alpha_1 \alpha_2, \mu_1 \mu_2} \times h_v^c(x). \end{aligned} \quad (15)$$

($\rho\lambda$ -b) $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 1$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\bar{\mathbf{3}}_F$ (\mathbf{A}), and we obtain a spin doublet ($1/2^+, 3/2^+$):

$$J_{1/2,+, \bar{\mathbf{3}}_F, 1, 1, \rho\lambda}(x) = \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \quad (16)$$

$$\times \left(g_t^{\mu_1 \mu_3} g_t^{\mu_2 \mu_4} + g_t^{\mu_2 \mu_3} g_t^{\mu_1 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{3/2,+, \bar{\mathbf{3}}_F, 1, 1, \rho\lambda}^\alpha(x) = \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right) \quad (17)$$

$$\times \left(\frac{1}{2} g_t^{\mu_1 \mu_3} g_t^{\mu_2 \alpha} + \frac{1}{2} g_t^{\mu_2 \mu_3} g_t^{\mu_1 \alpha} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_3 \alpha} \right) \times h_v^c(x).$$

($\rho\lambda$ -c) $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 2$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\bar{\mathbf{3}}_F$ (\mathbf{A}), and we obtain a spin doublet ($3/2^+, 5/2^+$). We failed to construct these currents.

($\rho\lambda$ -d) $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ with $s_l = 1$ (\mathbf{S}) and $j_l = 3$. Now the diquark has color $\bar{\mathbf{3}}_C$ (\mathbf{A}) and flavor $\bar{\mathbf{3}}_F$ (\mathbf{A}), and we obtain a spin doublet ($5/2^+, 7/2^+$):

$$J_{5/2,+, \bar{\mathbf{3}}_F, 3, 1, \rho\lambda}^{\alpha_1 \alpha_2}(x) \quad (18)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \Gamma_{\nu_1 \nu_2}^{\alpha_1 \alpha_2} \times \left(g_t^{\mu_1 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_3 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_2 \nu_2} g_t^{\mu_1 \mu_4} + g_t^{\mu_3 \nu_1} g_t^{\mu_1 \nu_2} g_t^{\mu_2 \mu_4} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{7/2,+, \bar{\mathbf{3}}_F, 3, 1, \rho\lambda}^{\alpha_1 \alpha_2 \alpha_3}(x) \quad (19)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^{aT}(x)] \mathbb{C} \gamma_{\mu_3}^t q^b(x) - q^{aT}(x) \mathbb{C} \gamma_{\mu_3}^t [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t q^b(x)] \right)$$

$$\times \Gamma_t^{\alpha_1 \alpha_2 \alpha_3, \mu_1 \mu_2 \mu_3} \times h_v^c(x).$$

We note that all these interpolating fields have been projected to $j = \frac{1}{2}/\frac{3}{2}/\frac{5}{2}/\frac{7}{2}$. Identical sum rules can be obtained using either $J_{[j_l-1/2], P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1 \dots \alpha_{|j_l-1/2|}}$ or $J_{[j_l+1/2], P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1 \dots \alpha_{j_l+1/2}}$ in the same doublet, both at the leading order and at the $O(1/m_Q)$ order [60–62, 64]. Hence, we only need to use one of them to perform QCD sum rule analyses.

There are altogether five baryon multiplets of $SU(3)$ flavor $\bar{\mathbf{3}}_F$, i.e., $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$, $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$. In the next section we shall use $J_{3/2,+, \bar{\mathbf{3}}_F, 2, 0, \rho\rho}^\alpha$, $J_{3/2,+, \bar{\mathbf{3}}_F, 2, 0, \lambda\lambda}^\alpha$, $J_{1/2,+, \bar{\mathbf{3}}_F, 1, 1, \rho\lambda}$ and $J_{7/2,+, \bar{\mathbf{3}}_F, 3, 1, \rho\lambda}^{\alpha_1 \alpha_2 \alpha_3}$ to perform QCD sum rule analyses. We shall further replace $\mathbf{6}_F$ by Σ_c , Ξ'_c , and Ω_c , and $\bar{\mathbf{3}}_F$ by Λ_c and Ξ_c to explicitly denote the quark contents inside, such as $J_{3/2,+, \Lambda_c, 2, 0, \lambda\lambda}^\alpha$ and $J_{3/2,+, \Xi_c, 2, 0, \lambda\lambda}^\alpha$ belonging to $[\Lambda_c, 2, 0, \lambda\lambda]$ and $[\Xi_c, 2, 0, \lambda\lambda]$, respectively:

$$J_{3/2,+, \Lambda_c, 2, 0, \lambda\lambda}^\alpha(x) \quad (20)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t u^{aT}(x)] \mathbb{C} \gamma_5 d^b(x) + 2[\mathcal{D}_{\mu_1}^t u^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t d^b(x)] + u^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t d^b(x)] \right)$$

$$\times \left(\frac{1}{2} g_t^{\mu_1 \alpha} g_t^{\mu_2 \mu_4} + \frac{1}{2} g_t^{\mu_2 \alpha} g_t^{\mu_1 \mu_4} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_4 \alpha} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x),$$

$$J_{3/2,+, \Xi_c, 2, 0, \lambda\lambda}^\alpha(x) \quad (21)$$

$$= \epsilon_{abc} \left([\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t u^{aT}(x)] \mathbb{C} \gamma_5 s^b(x) + 2[\mathcal{D}_{\mu_1}^t u^{aT}(x)] \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_2}^t s^b(x)] + u^{aT}(x) \mathbb{C} \gamma_5 [\mathcal{D}_{\mu_1}^t \mathcal{D}_{\mu_2}^t s^b(x)] \right)$$

$$\times \left(\frac{1}{2} g_t^{\mu_1 \alpha} g_t^{\mu_2 \mu_4} + \frac{1}{2} g_t^{\mu_2 \alpha} g_t^{\mu_1 \mu_4} - \frac{1}{3} g_t^{\mu_1 \mu_2} g_t^{\mu_4 \alpha} \right) \times \gamma_{\mu_4}^t \gamma_5 h_v^c(x).$$

III. SUM RULES AT THE LEADING ORDER

In the previous section we have partly classified the D -wave charmed baryon interpolating fields, and in this and next sections we use them to further perform QCD sum rule analyses. When classifying these fields, we have taken into account their inner structures by fixing their inner quantum numbers j_l , s_l , l_ρ , and l_λ . Although the physical state is probably a mixed state containing components with various inner quantum numbers, at the beginning we can always assume the state $|j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda\rangle$ exists, which has the quantum numbers j , P , F and the inner quantum numbers j_l , s_l , and $[\rho\rho/\lambda\lambda/\rho\lambda]$ in the $m_Q \rightarrow \infty$ limit. It belongs to the spin doublet of the spin $j = j_l \otimes s_Q = j_l \pm 1/2$

with $[F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda]$, and coupled by the interpolating field $J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2}}$ through

$$\langle 0 | J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2}} | j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda \rangle = f_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} u^{\alpha_1\cdots\alpha_{j-1/2}}, \quad (22)$$

where $f_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$ is the decay constant, and $u^{\alpha_1\cdots\alpha_j}$ is the relevant spinor. For examples, $u(x)$ and $u^\alpha(x)$ are the Dirac and Rarita-Schwinger spinors, respectively. Then the two-point correlation function can be written as

$$\begin{aligned} \Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2},\beta_1\cdots\beta_{j-1/2}}(\omega) &= i \int d^4x e^{ikx} \langle 0 | T [J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2}}(x) \bar{J}_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\beta_1\cdots\beta_{j-1/2}}(0)] | 0 \rangle \\ &= \mathbb{S}[g_t^{\alpha_1\beta_1} \cdots g_t^{\alpha_{j-1/2}\beta_{j-1/2}}] \frac{1+\not{p}}{2} \Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega) + \cdots, \end{aligned} \quad (23)$$

where ω is twice the external off-shell energy, $\omega = 2v \cdot k$, and $\mathbb{S}[\cdots]$ is used to denote symmetrization and subtracting the trace terms in the sets $(\alpha_1 \cdots \alpha_{j-1/2})$ and $(\beta_1 \cdots \beta_{j-1/2})$. The leading term $\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega)$ has been totally symmetrized and only contains the highest spin j component, while \cdots contains other spin components. We note that we have omitted the quantum numbers j and P simply because the two currents in the same doublet give identical sum rules at the leading order in the heavy quark limit.

At the hadron level the correlation function (24) can be simply written as

$$\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega) = \frac{2f_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^2}{2\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} - \omega} + \text{higher states}, \quad (24)$$

where $\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$ is the difference between the mass of the lowest-lying heavy baryon state and the heavy quark mass:

$$\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} \equiv \lim_{m_Q \rightarrow \infty} (m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} - m_Q). \quad (25)$$

At the quark and gluon level the correlation function (24) can be evaluated using the method of operator production expansion (OPE) [60–62, 64]. Using $J_{3/2,+, \Lambda_c, 2, 0, \lambda\lambda}^\alpha$ and $J_{3/2,+, \Xi_c, 2, 0, \lambda\lambda}^\alpha$ as examples, we insert Eqs. (20) and (21) into Eq. (24), perform the Borel transformation, and then obtain

$$\Pi_{\Lambda_c, 2, 0, \lambda\lambda}(\omega_c, T) = f_{\Lambda_c, 2, 0, \lambda\lambda}^2 e^{-2\bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda}/T} = \int_0^{\omega_c} \left[\frac{5}{145152\pi^4} \omega^9 - \frac{\langle g_s^2 GG \rangle}{1728\pi^4} \omega^5 \right] e^{-\omega/T} d\omega, \quad (26)$$

$$\begin{aligned} \Pi_{\Xi_c, 2, 0, \lambda\lambda}(\omega_c, T) &= f_{\Xi_c, 2, 0, \lambda\lambda}^2 e^{-2\bar{\Lambda}_{\Xi_c, 2, 0, \lambda\lambda}/T} \\ &= \int_{2m_s}^{\omega_c} \left[\frac{5}{145152\pi^4} \omega^9 - \frac{m_s^2}{672\pi^4} \omega^7 - \frac{m_s \langle \bar{q}q \rangle}{72\pi^2} \omega^5 + \frac{m_s \langle \bar{s}s \rangle}{48\pi^2} \omega^5 \right. \\ &\quad \left. - \frac{\langle g_s^2 GG \rangle}{1728\pi^4} \omega^5 + \frac{5m_s^2 \langle g_s^2 GG \rangle}{576\pi^4} \omega^3 - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{216\pi^2} \omega \right] e^{-\omega/T} d\omega. \end{aligned} \quad (27)$$

Sum rules for other currents are shown in Appendix B. We note that in our calculations we have used the software *Mathematica* with a package called *FeynCalc* [84]. The condensates and other parameters contained in these sum rules take the following values [1, 60–62, 64, 85–92]:

$$\begin{aligned} \langle \bar{q}q \rangle &= \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -(0.24 \text{ GeV})^3, \\ \langle \bar{s}s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q}q \rangle, \\ \langle \frac{\alpha_s}{\pi} GG \rangle &= 0.005 \pm 0.004 \text{ GeV}^4, \\ m_s &= 0.125 \text{ GeV}, \\ \langle g_s \bar{q} \sigma G q \rangle &= M_0^2 \times \langle \bar{q}q \rangle, \\ \langle g_s \bar{s} \sigma G s \rangle &= M_0^2 \times \langle \bar{s}s \rangle, \\ M_0^2 &= 0.8 \text{ GeV}^2. \end{aligned} \quad (28)$$

Finally, we differentiate Log[Eq. (26)] and Log[Eq. (27)] with respect to $[-2/T]$ to obtain $\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$:

$$\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T) = \frac{\frac{\partial}{\partial(-2/T)} \Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T)}{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T)}, \quad (29)$$

which can be further used to obtain $f_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$:

$$f_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T) = \sqrt{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T) \times e^{2\bar{\Lambda}_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T)/T}}. \quad (30)$$

There are two free parameters in Eq. (29), the Borel mass T and the threshold value ω_c . We have three criteria to constrain them. The first criterion is to require the high-order corrections to be less than 10%:

$$\text{Convergence (CVG)} \equiv \left| \frac{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\text{high-order}}(\infty, T)}{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\infty, T)} \right| \leq 10\%, \quad (31)$$

where $\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\text{high-order}}(\omega_c, T)$ is used to denote the high-order corrections, for example,

$$\Pi_{\Xi_c,2,0,\lambda\lambda}^{\text{high-order}}(\omega_c, T) = \int_{2m_s}^{\omega_c} \left[-\frac{m_s \langle \bar{q}q \rangle}{72\pi^2} \omega^5 + \frac{m_s \langle \bar{s}s \rangle}{48\pi^2} \omega^5 - \frac{\langle g_s^2 GG \rangle}{1728\pi^4} \omega^5 + \frac{5m_s^2 \langle g_s^2 GG \rangle}{576\pi^4} \omega^3 - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{216\pi^2} \omega \right] e^{-\omega/T} d\omega. \quad (32)$$

The second criterion is to require the pole contribution (PC) to be larger than 10%:

$$\text{PC} \equiv \frac{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega_c, T)}{\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\infty, T)} \geq 10\%. \quad (33)$$

Altogether we obtain an interval $T_{\min} < T < T_{\max}$ for a fixed threshold value ω_c .

The small pole contribution used in Eq. (33) is mathematically due to the large powers of s in the spectral function, which makes the suppression of the Borel transformation on the continuum not so effective. For example, see Ref. [93] where the pole contribution of the $d^*(2380)$ is only about 0.0002 due to the large power of s in its spectral function. However, actually we do not need a pole which is significant in the whole energy space, but just need it to be dominant inside our working region. Such a pole can be found as if the mass prediction does not depend on the other free parameter, the threshold value ω_c . Hence, the third criterion is to require the dependence of $m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$ (mass of the heavy baryon state) on the threshold value ω_c to be weak, which will be discussed in detail in Sec. V. At the same time we shall also check the dependence of $m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$ on the Borel mass T .

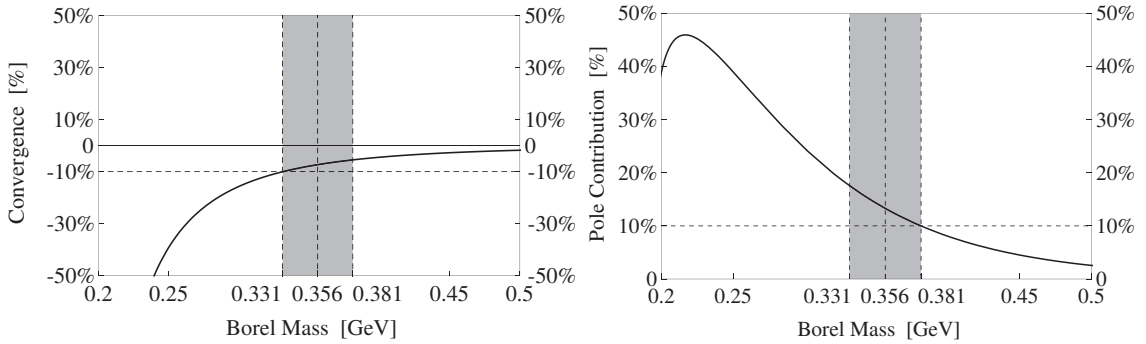


FIG. 2: In the left panel we show the variation of CVG, defined in Eq. (31), as a function of the Borel mass T . In the right panel we show the variation of PC, defined in Eq. (33), as a function of the Borel mass T , where the threshold value is chosen to be $\omega_c = 2.5$ GeV. The current $J_{3/2,+, \Lambda_c, 2, 0, \lambda\lambda}^\alpha$ is used here.

Still using the current $J_{3/2,+, \Lambda_c, 2, 0, \lambda\lambda}^\alpha$ as an example, we show the variations of CVG and PC, as defined in Eqs. (31) and (33), with respect to the Borel mass T in Fig. 2, and the variations of $\bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda}$ and $f_{\Lambda_c, 2, 0, \lambda\lambda}$ with respect to T in Fig. 3, where ω_c is chosen to be 2.5 GeV. Now the Borel window is $0.331 \text{ GeV} < T < 0.381 \text{ GeV}$, and we obtain the following numerical results:

$$\begin{aligned} \bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda} &= 1.113 \text{ GeV}, \\ f_{\Lambda_c, 2, 0, \lambda\lambda} &= 0.012 \text{ GeV}^5, \end{aligned} \quad (34)$$

where the central values are obtained by choosing $T = 0.356 \text{ GeV}$ and $\omega_c = 2.5 \text{ GeV}$.

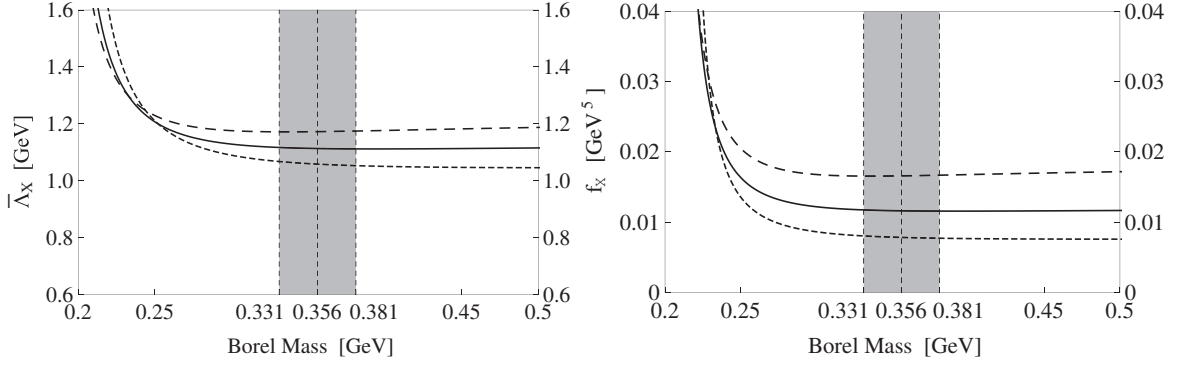


FIG. 3: The variations of $\bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda}$ (left) and $f_{\Lambda_c, 2, 0, \lambda\lambda}$ (right) with respect to the Borel mass T , when $J_{3/2, +, \Lambda_c, 2, 0, \lambda\lambda}^\alpha$ is used. The short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 2.3, 2.5$, and 2.7 GeV, respectively.

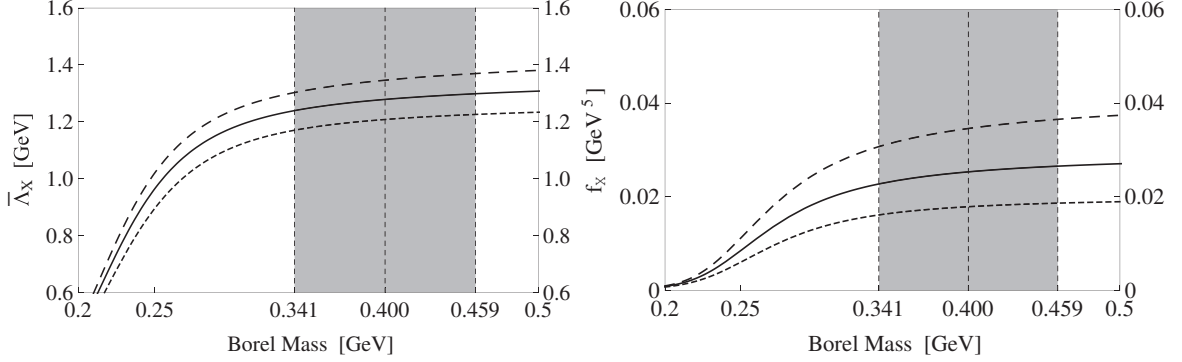


FIG. 4: The variations of $\bar{\Lambda}_{\Xi_c, 2, 0, \lambda\lambda}$ (left) and $f_{\Xi_c, 2, 0, \lambda\lambda}$ (right) with respect to the Borel mass T , when $J_{3/2, +, \Xi_c, 2, 0, \lambda\lambda}^\alpha$ is used. The short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 2.8, 3.0$, and 3.2 GeV, respectively.

We also show the variations of $\bar{\Lambda}_{\Xi_c, 2, 0, \lambda\lambda}$ and $f_{\Xi_c, 2, 0, \lambda\lambda}$ with respect to T in Fig. 4, where ω_c is chosen to be 3.0 GeV. From these figures, we find the Borel window $0.341 \text{ GeV} < T < 0.459 \text{ GeV}$, and obtain the following numerical results:

$$\begin{aligned}\bar{\Lambda}_{\Xi_c, 2, 0, \lambda\lambda} &= 1.279 \text{ GeV}, \\ f_{\Xi_c, 2, 0, \lambda\lambda} &= 0.025 \text{ GeV}^5,\end{aligned}\tag{35}$$

where the central values are obtained by choosing $T = 0.400 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$.

IV. SUM RULES AT THE ORDER $\mathcal{O}(1/m_Q)$

In this section we work up to the order $\mathcal{O}(1/m_Q)$ based on the HQET Lagrangian [62, 64]:

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S},\tag{36}$$

where \mathcal{K} is the operator of nonrelativistic kinetic energy, and \mathcal{S} is the Pauli term describing the chromomagnetic interaction:

$$\begin{aligned}\mathcal{K} &= \bar{h}_v (iD_t)^2 h_v, \\ \mathcal{S} &= \frac{g}{2} C_{\text{mag}}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v.\end{aligned}\tag{37}$$

Here $C_{\text{mag}}(m_Q/\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ with $\beta_0 = 11 - 2n_f/3$.

The correlation function at the hadron level, Eq. (24), can be written up to the order $\mathcal{O}(1/m_Q)$ as

$$\begin{aligned}\Pi(\omega)_{pole} &= \frac{2(f + \delta f)^2}{2(\bar{\Lambda} + \delta m) - \omega} \\ &= \frac{2f^2}{2\bar{\Lambda} - \omega} - \frac{4\delta m f^2}{(2\bar{\Lambda} - \omega)^2} + \frac{4f\delta f}{2\bar{\Lambda} - \omega},\end{aligned}\quad (38)$$

where $\delta m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$ is the correction to the mass $m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}$, and can be evaluated using the three-point correlation functions:

$$\begin{aligned}\delta_O \Pi_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1 \dots \alpha_{j-1/2}, \beta_1 \dots \beta_{j-1/2}}(\omega, \omega') &= i^2 \int d^4x d^4y e^{ik \cdot x - ik' \cdot y} \times \langle 0 | T [J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1 \dots \alpha_{j-1/2}}(x) O(0) \bar{J}_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\beta_1 \dots \beta_{j-1/2}}(y)] | 0 \rangle \\ &= \mathbb{S}[g_t^{\alpha_1 \beta_1} \dots g_t^{\alpha_{j-1/2} \beta_{j-1/2}}] \delta_O \Pi_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega),\end{aligned}\quad (39)$$

where $O = \mathcal{K}$ or \mathcal{S} . Based on the Lagrangian (36), these correlation functions can be written at the hadron level as

$$\delta_{\mathcal{K}} \Pi(\omega, \omega')_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} = \frac{2f^2 K_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \frac{2f^2 G_{\mathcal{K}}(\omega')}{2\bar{\Lambda} - \omega} + \frac{2f^2 G_{\mathcal{K}}(\omega)}{2\bar{\Lambda} - \omega'}, \quad (40)$$

$$\delta_{\mathcal{S}} \Pi(\omega, \omega')_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} = \frac{2d_M f^2 \Sigma_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}}{(2\bar{\Lambda} - \omega)(2\bar{\Lambda} - \omega')} + \frac{2d_M f^2 G_{\mathcal{S}}(\omega')}{2\bar{\Lambda} - \omega} + \frac{2d_M f^2 G_{\mathcal{S}}(\omega)}{2\bar{\Lambda} - \omega'}, \quad (41)$$

where the following definitions have been used:

$$\begin{aligned}K_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} &\equiv \langle j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda | \bar{h}_v (iD_{\perp})^2 h_v | j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda \rangle, \\ d_M \Sigma_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} &\equiv \langle j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda | \frac{g}{2} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v | j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda \rangle, \\ d_M &\equiv d_{j,j_l}, \\ d_{j_l-1/2,j_l} &= 2j_l + 2, \\ d_{j_l+1/2,j_l} &= -2j_l.\end{aligned}\quad (42)$$

Then we fix $\omega = \omega'$ and use Eqs. (38), (40), and (41) to obtain

$$\delta m_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} = -\frac{1}{4m_Q} (K_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda} + d_M C_{mag} \Sigma_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}). \quad (43)$$

From this equation we find that only the term $\mathcal{S}(\Sigma_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda})$ can cause a mass splitting within the same doublet.

The three-point correlation functions defined in Eq. (39) can also be evaluated at the quark and gluon level using the method of operator product expansion [62, 64]. Still using the currents $J_{3/2,+,\Lambda_c,2,0,\lambda\lambda}^{\alpha}$ and $J_{3/2,+,\Xi_c,2,0,\lambda\lambda}^{\alpha}$ as examples, we insert Eqs. (20) and (21) into Eqs. (39), make a double Borel transformation for both ω and ω' , take the two Borel parameters to be equal, and then obtain:

$$f_{\Lambda_c,2,0,\lambda\lambda}^2 K_{\Lambda_c,2,0,\lambda\lambda} e^{-2\bar{\Lambda}_{\Lambda_c,2,0,\lambda\lambda}/T} = \int_0^{\omega_c} \left[-\frac{127}{10644480\pi^4} \omega^{11} - \frac{\langle g_s^2 GG \rangle}{17280\pi^4} \omega^7 \right] e^{-\omega/T} d\omega, \quad (44)$$

$$f_{\Lambda_c,2,0,\lambda\lambda}^2 \Sigma_{\Lambda_c,2,0,\lambda\lambda} e^{-2\bar{\Lambda}_{\Lambda_c,2,0,\lambda\lambda}/T} = \int_0^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{24192\pi^4} \omega^7 \right] e^{-\omega/T} d\omega, \quad (45)$$

$$f_{\Xi_c,2,0,\lambda\lambda}^2 K_{\Xi_c,2,0,\lambda\lambda} e^{-2\bar{\Lambda}_{\Xi_c,2,0,\lambda\lambda}/T} \quad (46)$$

$$\begin{aligned}&= \int_{2m_s}^{\omega_c} \left[-\frac{127}{10644480\pi^4} \omega^{11} + \frac{307m_s^2}{483840\pi^4} \omega^9 + \frac{37m_s \langle \bar{q}q \rangle}{5040\pi^2} \omega^7 - \frac{233m_s \langle \bar{s}s \rangle}{20160\pi^2} \omega^7 \right. \\ &\quad \left. - \frac{\langle g_s^2 GG \rangle}{17280\pi^4} \omega^7 - \frac{1019m_s^2 \langle g_s^2 GG \rangle}{184320\pi^4} \omega^5 - \frac{13m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{648\pi^2} \omega^3 + \frac{67m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{2304\pi^2} \omega^3 \right] e^{-\omega/T} d\omega,\end{aligned}$$

$$f_{\Xi_c,2,0,\lambda\lambda}^2 \Sigma_{\Xi_c,2,0,\lambda\lambda} e^{-2\bar{\Lambda}_{\Xi_c,2,0,\lambda\lambda}/T} \quad (47)$$

$$= \int_{2m_s}^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{24192\pi^4} \omega^7 - \frac{m_s^2 \langle g_s^2 GG \rangle}{1536\pi^4} \omega^5 + \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{864\pi^2} \omega^3 \right] e^{-\omega/T} d\omega,$$

Sum rules for other currents are shown in Appendix B.

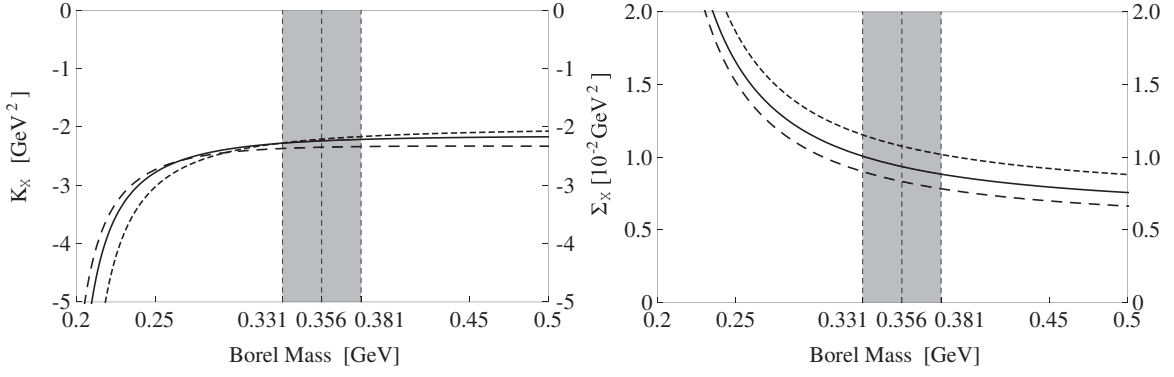


FIG. 5: The variations of $K_{\Lambda_c,2,0,\lambda\lambda}$ (left) and $\Sigma_{\Lambda_c,2,0,\lambda\lambda}$ (right) with respect to the Borel mass T , when $J_{3/2,+,\Lambda_c,2,0,\lambda\lambda}^\alpha$ is used. The short-dashed, solid and long-dashed curves are obtained by fixing $\omega_c = 2.3, 2.5$ and 2.7 GeV, respectively.

Finally, we obtain $K_{\Lambda_c,2,0,\lambda\lambda}$ and $\Sigma_{\Lambda_c,2,0,\lambda\lambda}$ by simply dividing Eqs. (44) and (45) by Eq. (26). Their variations are shown in Fig. 5 with respect to the Borel mass T . We find their dependence on T is weak in the Borel window $0.331 \text{ GeV} < T < 0.381 \text{ GeV}$, and obtain the following numerical results:

$$\begin{aligned} K_{\Lambda_c,2,0,\lambda\lambda} &= -2.239 \text{ GeV}^2, \\ \Sigma_{\Lambda_c,2,0,\lambda\lambda} &= 0.014 \text{ GeV}^2, \end{aligned} \quad (48)$$

where the central values are obtained by choosing $T = 0.356 \text{ GeV}$ and $\omega_c = 2.5 \text{ GeV}$.

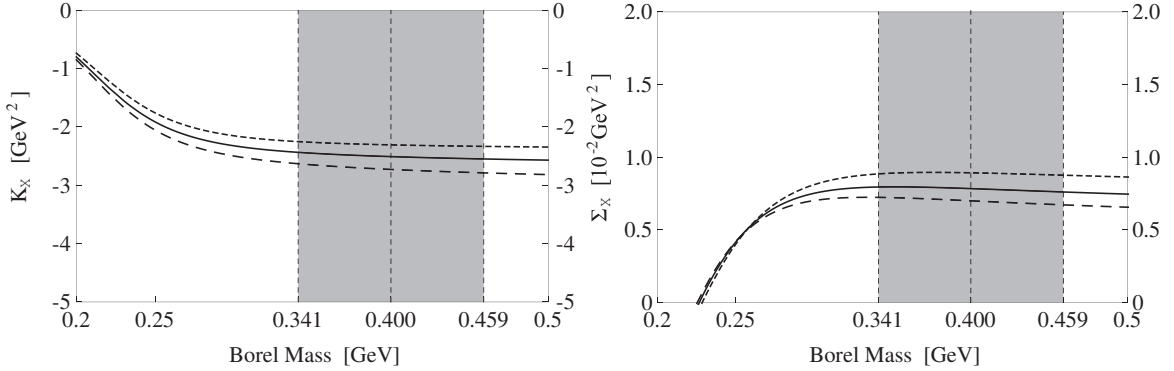


FIG. 6: The variations of $K_{\Xi_c,2,0,\lambda\lambda}$ (left) and $\Sigma_{\Xi_c,2,0,\lambda\lambda}$ (right) with respect to the Borel mass T , when $J_{3/2,+,\Xi_c,2,0,\lambda\lambda}^\alpha$ is used. The short-dashed, solid and long-dashed curves are obtained by fixing $\omega_c = 2.8, 3.0$ and 3.2 GeV, respectively.

We also obtain $K_{\Xi_c,2,0,\lambda\lambda}$ and $\Sigma_{\Xi_c,2,0,\lambda\lambda}$ by simply dividing Eqs. (46) and (47) by Eq. (27), and show their variations in Fig. 6 with respect to the Borel mass T . We find their dependence on T is weak in the Borel window $0.341 \text{ GeV} < T < 0.459 \text{ GeV}$, and obtain the following numerical results:

$$\begin{aligned} K_{\Xi_c,2,0,\lambda\lambda} &= -2.508 \text{ GeV}^2, \\ \Sigma_{\Xi_c,2,0,\lambda\lambda} &= 0.008 \text{ GeV}^2, \end{aligned} \quad (49)$$

where the central values are obtained by choosing $T = 0.400 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$.

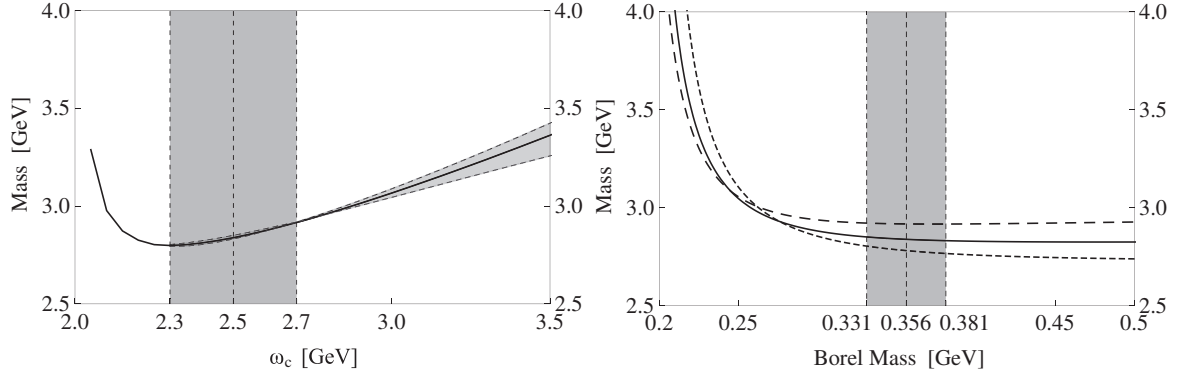


FIG. 7: Variations of $m_{\Lambda_c(5/2^+)}$ with respect to the threshold value ω_c (left) and the Borel mass T (right), calculated using the charmed baryon doublet $[\Lambda_c, 2, 0, \lambda\lambda]$. In the left panel, the shady band is obtained by changing T inside Borel windows. The mass curves have minimum against ω_c around 2.2 GeV, where the ω_c dependence of the mass prediction is the weakest. However, at this point there does not exist any non-vanishing working region of the Borel mass T . We find that there exist non-vanishing working regions of T as long as $\omega_c \geq 2.3$ GeV, and the ω_c dependence is still weak and acceptable in the region $2.3 \text{ GeV} < \omega_c < 2.7 \text{ GeV}$. The results for $\omega_c < 2.3$ GeV are also shown, for which cases we choose the Borel mass T when the PC, as defined in Eq. (34), is around 10%. In the right figure, the short-dashed, solid and long-dashed curves are obtained by fixing $\omega_c = 2.3, 2.5$ and 2.7 GeV, respectively.

V. NUMERICAL RESULTS AND DISCUSSIONS

Combining the results obtained in Sec. III and Sec. IV, we obtain the masses of the heavy baryon doublet $[\Lambda_c, 2, 0, \lambda\lambda]$ satisfying:

$$\begin{aligned} m_{\Lambda_c(3/2^+)} &= m_c + \bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda} - \frac{1}{4m_c} [K_{\Lambda_c, 2, 0, \lambda\lambda} + d_{3/2, 2} \Sigma_{\Lambda_c, 2, 0, \lambda\lambda}], \\ m_{\Lambda_c(5/2^+)} &= m_c + \bar{\Lambda}_{\Lambda_c, 2, 0, \lambda\lambda} - \frac{1}{4m_c} [K_{\Lambda_c, 2, 0, \lambda\lambda} + d_{5/2, 2} \Sigma_{\Lambda_c, 2, 0, \lambda\lambda}]. \end{aligned} \quad (50)$$

After inserting $d_{3/2, 2} = 6$ and $d_{5/2, 2} = -4$, we arrive at:

$$\begin{aligned} \frac{1}{10} (4m_{\Lambda_c(3/2^+)} + 6m_{\Lambda_c(5/2^+)}) &= m_c + 1.113 \text{ GeV} - \frac{1}{4m_c} [-2.239 \text{ GeV}^2], \\ m_{\Lambda_c(5/2^+)} - m_{\Lambda_c(3/2^+)} &= \frac{1}{4m_c} \times 10 \times [0.014 \text{ GeV}^2], \end{aligned} \quad (51)$$

where $\Lambda_c(3/2^+)$ and $\Lambda_c(5/2^+)$ are the two baryons contained in this doublet. Clearly, the $\mathcal{O}(1/m_Q)$ corrections can not be neglected. Then we use the PDG value $m_c = 1.275 \pm 0.025 \text{ GeV}$ [1] for the charm quark mass in the $\overline{\text{MS}}$ scheme to obtain numerical results:

$$\begin{aligned} m_{\Lambda_c(3/2^+)} &= 2.81 \text{ GeV}, \\ m_{\Lambda_c(5/2^+)} &= 2.84 \text{ GeV}, \\ m_{\Lambda_c(5/2^+)} - m_{\Lambda_c(3/2^+)} &= 28 \text{ MeV}. \end{aligned} \quad (52)$$

These values are obtained for $\omega_c = 2.5 \text{ GeV}$. We change the threshold value ω_c and redo the same procedures. We note that our third criterion is to require the dependence of $m_{j, P, F, j_l, s_l, \rho\rho/\lambda\lambda/\rho\lambda}$ (mass of the heavy baryon state) on this parameter ω_c to be weak. Accordingly, we show the variation of $m_{\Lambda_c(5/2^+)}$ with respect to ω_c in the left panel of Fig. 7 in a large region $2.0 \text{ GeV} < \omega_c < 3.5 \text{ GeV}$. The mass curves have minimum against ω_c around 2.2 GeV, where the ω_c dependence of the mass prediction is the weakest. However, at this point we apply the two criteria on the Borel mass T (see discussions in Sec. III) but can not obtain any non-vanishing working region of T . We find that there exist non-vanishing working regions of T as long as $\omega_c \geq 2.3 \text{ GeV}$, and the ω_c dependence is still weak and acceptable in the region $2.3 \text{ GeV} < \omega_c < 2.7 \text{ GeV}$. Hence, we choose $2.3 \text{ GeV} < \omega_c < 2.7 \text{ GeV}$ and $0.331 \text{ GeV} < T < 0.381 \text{ GeV}$

as our working regions, and obtain the following numerical results for the baryon doublet $[\Lambda_c, 2, 0, \lambda\lambda]$:

$$\begin{aligned} m_{\Lambda_c(3/2^+)} &= 2.81^{+0.33}_{-0.18} \text{ GeV}, \\ m_{\Lambda_c(5/2^+)} &= 2.84^{+0.37}_{-0.20} \text{ GeV}, \\ m_{\Lambda_c(5/2^+)} - m_{\Lambda_c(3/2^+)} &= 28^{+45}_{-24} \text{ MeV}, \end{aligned} \quad (53)$$

whose central values correspond to $T = 0.356 \text{ GeV}$ and $\omega_c = 2.5 \text{ GeV}$, and the uncertainties are due to the Borel mass T , the threshold value ω_c , the charm quark mass m_c and the quark and gluon condensates. We also show the variation of $m_{\Lambda_c(5/2^+)}$ with respect to the Borel mass T in the right panel of Fig. 7, in a broad region $0.2 \text{ GeV} < T < 0.5 \text{ GeV}$, where these curves are more stable inside the Borel window $0.331 \text{ GeV} < T < 0.381 \text{ GeV}$. The mass of the $\Lambda_c(5/2^+)$ in the doublet $[\Lambda_c, 2, 0, \lambda\lambda]$ is consistent with the mass of the $\Lambda_c(2880)$ [1]:

$$m_{\Lambda_c(2880), 5/2^+}^{\text{exp}} = 2881.53 \pm 0.35 \text{ MeV}, \quad (54)$$

and supports it to be a D -wave charmed baryon of $J^P = 5/2^+$. Our result further suggests that the $\Lambda_c(2880)$ of $J^P = 5/2^+$ has a partner state, the $\Lambda_c(3/2^+)$ of $J^P = 3/2^+$. Its mass is $2.81^{+0.33}_{-0.18} \text{ GeV}$, and the mass difference between it and the $\Lambda_c(2880)$ is $28^{+45}_{-24} \text{ MeV}$. We note that there are large theoretical uncertainties in our results for the masses of the heavy baryons, but their differences within the same doublet are produced with much less theoretical uncertainty because they do not depend much on the charm quark mass and the threshold value [47, 48].

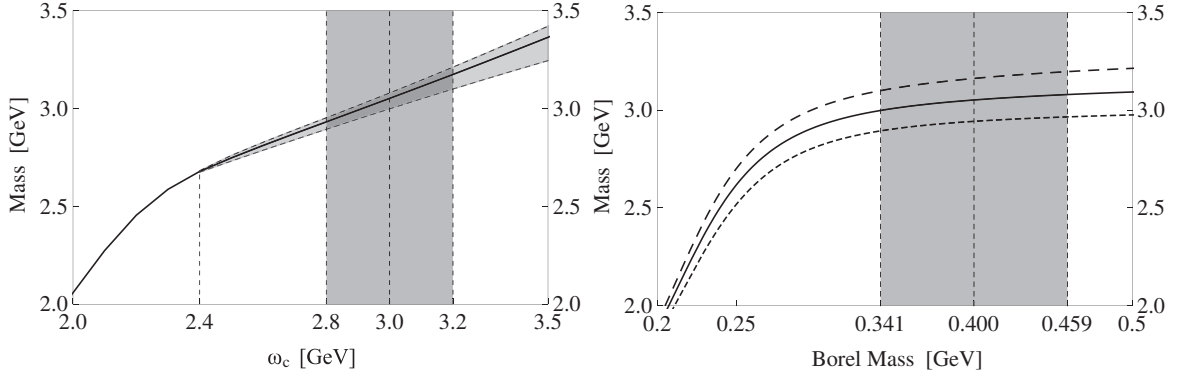


FIG. 8: Variations of $m_{\Xi_c(5/2^+)}$ with respect to the threshold value ω_c (left) and the Borel mass T (right), calculated using the charmed baryon doublet $[\Xi_c, 2, 0, \lambda\lambda]$. In the left panel, the shady band is obtained by changing T inside Borel windows, which exist as long as $\omega_c \geq 2.4 \text{ GeV}$. We properly fine-tune the threshold value ω_c to be around 3.0 GeV so that $\omega_c(\Xi_c(5/2^+)) - \omega_c(\Lambda_c(5/2^+)) = 0.5 \text{ GeV}$, which value is the same as those used in our previous studies on P -wave heavy baryons [47, 48]. In the right figure, the short-dashed, solid and long-dashed curves are obtained by fixing $\omega_c = 2.8, 3.0$ and 3.2 GeV , respectively.

We follow the same procedures to study the baryon doublet $[\Xi_c, 2, 0, \lambda\lambda]$, and show the variation of $m_{\Xi_c(5/2^+)}$ with respect to the threshold value ω_c in the left panel of Fig. 8. Different from the case of $m_{\Lambda_c(5/2^+)}$, the mass curves do not have minimum against ω_c , but the ω_c dependence of the mass prediction is still not strong when $\omega_c > 2.5 \text{ GeV}$ where there exist Borel windows. We properly fine-tune the threshold value ω_c to be around 3.0 GeV so that $\omega_c(\Xi_c(5/2^+)) - \omega_c(\Lambda_c(5/2^+)) = 0.5 \text{ GeV}$, which value is the same as those used in our previous studies on P -wave heavy baryons [47, 48]. Together we choose $2.8 \text{ GeV} < \omega_c < 3.2 \text{ GeV}$ and $0.341 \text{ GeV} < T < 0.459 \text{ GeV}$ as our working regions, and obtain the following numerical results for the baryon doublet $[\Xi_c, 2, 0, \lambda\lambda]$:

$$\begin{aligned} m_{\Xi_c(3/2^+)} &= 3.04^{+0.15}_{-0.15} \text{ GeV}, \\ m_{\Xi_c(5/2^+)} &= 3.05^{+0.15}_{-0.16} \text{ GeV}, \\ m_{\Xi_c(5/2^+)} - m_{\Xi_c(3/2^+)} &= 15^{+16}_{-13} \text{ MeV}, \end{aligned} \quad (55)$$

whose central values correspond to $T = 0.400 \text{ GeV}$ and $\omega_c = 3.0 \text{ GeV}$. We also show the variation of $m_{\Xi_c(5/2^+)}$ with respect to the Borel mass T in the right panel of Fig. 8, where these curves are stable inside the Borel window $0.341 \text{ GeV} < T < 0.459 \text{ GeV}$. The masses of the $\Xi_c(3/2^+)$ and $\Xi_c(5/2^+)$ in the doublet $[\Xi_c, 2, 0, \lambda\lambda]$ are consistent with the

masses of the $\Xi_c(3055)$ and $\Xi_c(3080)$ [1] as well as their difference:

$$\begin{aligned} m_{\Xi_c(3055)+}^{\text{exp}} &= 3055.1 \pm 1.7 \text{ MeV}, \\ m_{\Xi_c(3080)+}^{\text{exp}} &= 3076.94 \pm 0.28 \text{ MeV}, m_{\Xi_c(3080)^0}^{\text{exp}} = 3079.9 \pm 1.4 \text{ MeV}, \\ m_{\Xi_c(3080)+}^{\text{exp}} - m_{\Xi_c(3055)+}^{\text{exp}} &= 21.8 \pm 1.7 \text{ MeV}. \end{aligned} \quad (56)$$

This suggests that the $\Xi_c(3055)$ and $\Xi_c(3080)$ have quantum number $J^P = 3/2^+$ and $5/2^+$, respectively, which assignments have been proposed or discussed in detail in Refs. [15–17].

TABLE I: Masses of the D -wave charmed baryons obtained using the baryon doublets $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$, $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$. As discussed at the end of Sec. II, a) for the baryon doublet $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ containing $\Lambda_c(5/2^+, 7/2^+)$ and $\Xi_c(5/2^+, 7/2^+)$, we only evaluate their average masses $\frac{1}{14}(6m_{\Lambda_c(5/2^+)} + 8m_{\Lambda_c(7/2^+)})$ and $\frac{1}{14}(6m_{\Xi_c(5/2^+)} + 8m_{\Xi_c(7/2^+)})$; b) for the baryon doublet $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 2$), we estimate their masses by simply averaging between $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 1$) and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 3$). We assume that free parameters ω_c in the same multiplet satisfy the relation $\omega_c(\Xi_c) - \omega_c(\Lambda_c) = 0.5 \text{ GeV}$, except for the $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$.

Multiplets	B	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV ⁵)	K (GeV ²)	Σ (GeV ²)	Baryons (j^P)	Mass (GeV)	Difference (MeV)
$[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$	Λ_c	2.5	$0.331 < T < 0.381$	1.113	0.012	-2.239	0.014	$\Lambda_c(3/2^+)$	$2.81^{+0.33}_{-0.18}$	28^{+45}_{-24}
								$\Lambda_c(5/2^+)$	$2.84^{+0.37}_{-0.20}$	
	Ξ_c	3.0	$0.341 < T < 0.459$	1.279	0.025	-2.508	0.008	$\Xi_c(3/2^+)$	$3.04^{+0.15}_{-0.15}$	15^{+16}_{-13}
								$\Xi_c(5/2^+)$	$3.05^{+0.15}_{-0.16}$	
$[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$	Λ_c	3.4	$0.358 < T < 0.499$	1.650	0.065	-1.742	0.011	$\Lambda_c(3/2^+)$	$3.25^{+1.72}_{-0.28}$	22^{+120}_{-20}
								$\Lambda_c(5/2^+)$	$3.28^{+1.83}_{-0.30}$	
	Ξ_c	3.9	$0.502 < T < 0.591$	1.723	0.10	-1.308	0.006	$\Xi_c(3/2^+)$	$3.25^{+0.16}_{-0.14}$	11^{+16}_{-9}
								$\Xi_c(5/2^+)$	$3.26^{+0.17}_{-0.15}$	
$[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$	Λ_c	3.0	$0.397 < T < 0.457$	1.335	0.044	-2.116	0.006	$\Lambda_c(1/2^+)$	$3.02^{+0.19}_{-0.14}$	7^{+11}_{-6}
								$\Lambda_c(3/2^+)$	$3.03^{+0.20}_{-0.14}$	
	Ξ_c	4.0	$0.526 < T < 0.599$	1.887	0.21	-2.954	0.003	$\Xi_c(1/2^+)$	$3.74^{+0.14}_{-0.13}$	3^{+3}_{-3}
								$\Xi_c(3/2^+)$	$3.74^{+0.14}_{-0.13}$	
$[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ (estimated)	Λ_c	-	-	-	-	-	-	$\Lambda_c(3/2^+)$	~ 3.20	-
								$\Lambda_c(5/2^+)$		
	Ξ_c	-	-	-	-	-	-	$\Xi_c(3/2^+)$	~ 3.76	-
								$\Xi_c(5/2^+)$		
$[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ (simplified)	Λ_c	3.6	$0.496 < T < 0.542$	1.628	0.022	-2.939	-	$\Lambda_c(5/2^+)$	$3.48^{+0.33}_{-0.18}$	-
								$\Lambda_c(7/2^+)$		
	Ξ_c	4.1	$0.556 < T < 0.609$	1.920	0.045	-3.105	-	$\Xi_c(5/2^+)$	$3.80^{+0.20}_{-0.16}$	-
								$\Xi_c(7/2^+)$		

We also study the other four baryon doublets, $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$. Two important notes are:

1. It is too complicated to directly use $J_{7/2,+, \bar{\mathbf{3}}_F, 3, 1, \rho\lambda}^{\alpha_1 \alpha_2 \alpha_3}$, defined in Eq. (19), to perform QCD sum rule analyses, so we shall use its simplified version without the projection operator $\Gamma^{\alpha_1 \alpha_2 \alpha_3, \mu_1 \mu_2 \mu_3}$:

$$J_{7/2,+, \bar{\mathbf{3}}_F, 3, 1, \rho\lambda}^{\alpha_1 \alpha_2 \alpha_3}(x) = \mathbb{S}'[i^2 \epsilon_{abc} \left([\mathcal{D}^{\alpha_1} \mathcal{D}^{\alpha_2} q^{aT}(x)] \mathbb{C} \gamma_{\alpha_3} q^b(x) - q^{aT}(x) \mathbb{C} \gamma_{\alpha_3} [\mathcal{D}^{\alpha_1} \mathcal{D}^{\alpha_2} q^b(x)] \right) \times h_v^c(x)], \quad (57)$$

where $\mathcal{S}'[\dots]$ is used to denote symmetrization and subtracting the trace terms in the sets $(\alpha_1 \dots \alpha_3)$. Using this current, we can well calculate sum rules at the leading order as well as the \mathcal{K} correction ($K_{\bar{\mathbf{3}}_F, 3, 1, \rho\lambda}$) at the order $\mathcal{O}(1/m_Q)$, but the \mathcal{S} correction ($\Sigma_{\bar{\mathbf{3}}_F, 3, 1, \rho\lambda}$) at the order $\mathcal{O}(1/m_Q)$ can not be evaluated.

2. Because we failed to construct the two currents belonging to the baryon doublet $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ with $s_l = 1$ and $j_l = 2$, we shall estimate their masses by averaging between $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 1$) and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 3$), weighted by the spin-orbital splittings:

$$l_\rho \otimes s_l = \frac{1}{2}(j_l(j_l + 1) - l_\rho(l_\rho + 1) - s_l(s_l + 1)) = \frac{1}{2}(j_l(j_l + 1) - 4). \quad (58)$$

Hence, we obtain

$$\text{Mass}(j_l = 2) = \frac{3}{5} \times \text{Mass}(j_l = 1) + \frac{2}{5} \times \text{Mass}(j_l = 3). \quad (59)$$

However, their obtained results are difficult to explain the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$ at the same time:

1. The baryon doublet $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$ contains $\Lambda_c(3/2^+, 5/2^+)$ and $\Xi_c(3/2^+, 5/2^+)$. We use them to perform QCD sum rule analyses, and show variations of $m_{\Lambda_c(5/2^+)}$ and $m_{\Xi_c(5/2^+)}$ with respect to the threshold value ω_c in Fig. 9. The obtained masses are listed in Table I:

$$\begin{aligned} m_{\Lambda_c(3/2^+)} &= 3.25_{-0.28}^{+1.72} \text{ GeV}, m_{\Lambda_c(5/2^+)} = 3.28_{-0.30}^{+1.83} \text{ GeV}, \Delta m = 22_{-20}^{+120} \text{ MeV}, \\ m_{\Xi_c(3/2^+)} &= 3.25_{-0.14}^{+0.16} \text{ GeV}, m_{\Xi_c(5/2^+)} = 3.26_{-0.15}^{+0.17} \text{ GeV}, \Delta m = 11_{-9}^{+16} \text{ MeV}, \end{aligned} \quad (60)$$

whose values are significantly larger than the masses of the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$.

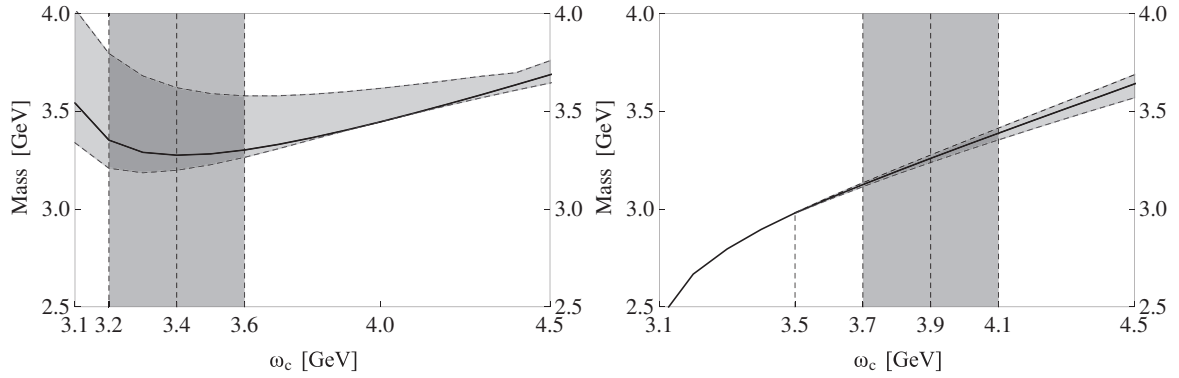


FIG. 9: Variations of $m_{\Lambda_c(5/2^+)}$ (left) and $m_{\Xi_c(5/2^+)}$ (right) with respect to the threshold value ω_c , calculated using the charmed baryon doublet $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$. The shady band is obtained by changing T inside Borel windows, which exist as long as $\omega_c \geq 3.1$ GeV (left) and $\omega_c \geq 3.5$ GeV (right). In the left panel we choose ω_c to be around 3.4 GeV, where the mass curves have minimum against it. In the right panel we properly fine-tune ω_c to be around 3.9 GeV so that $\omega_c(\Xi_c(5/2^+)) - \omega_c(\Lambda_c(5/2^+)) = 0.5$ GeV.

2. The baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$ contains $\Lambda_c(1/2^+, 3/2^+)$ and $\Xi_c(1/2^+, 3/2^+)$. This doublet does not contain any baryon of $J^P = 5/2^+$. We use them to perform QCD sum rule analyses, and show variations of $m_{\Lambda_c(3/2^+)}$ and $m_{\Xi_c(3/2^+)}$ with respect to the threshold value ω_c in Fig. 10. The obtained masses are listed in Table I:

$$\begin{aligned} m_{\Lambda_c(1/2^+)} &= 3.02_{-0.14}^{+0.19} \text{ GeV}, m_{\Lambda_c(3/2^+)} = 3.03_{-0.14}^{+0.20} \text{ GeV}, \Delta m = 7_{-6}^{+11} \text{ MeV}, \\ m_{\Xi_c(1/2^+)} &= 3.74_{-0.13}^{+0.14} \text{ GeV}, m_{\Xi_c(3/2^+)} = 3.74_{-0.13}^{+0.14} \text{ GeV}, \Delta m = 3_{-3}^{+3} \text{ MeV}. \end{aligned} \quad (61)$$

3. The baryon doublet $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ contains $\Lambda_c(5/2^+, 7/2^+)$ and $\Xi_c(5/2^+, 7/2^+)$. We use them to perform QCD sum rule analyses. As discussed at the end of Sec. II, we can only calculate their average masses. Using the following formulae

$$\begin{aligned} m_{\Lambda_c(5/2^+)} &= m_c + \bar{\Lambda}_{\Lambda_c, 3, 1, \rho\lambda} - \frac{1}{4m_c} [K_{\Lambda_c, 3, 1, \rho\lambda} + d_{5/2, 3} \Sigma_{\Lambda_c, 3, 1, \rho\lambda}], \\ m_{\Lambda_c(7/2^+)} &= m_c + \bar{\Lambda}_{\Lambda_c, 3, 1, \rho\lambda} - \frac{1}{4m_c} [K_{\Lambda_c, 3, 1, \rho\lambda} + d_{7/2, 3} \Sigma_{\Lambda_c, 3, 1, \rho\lambda}], \end{aligned} \quad (62)$$

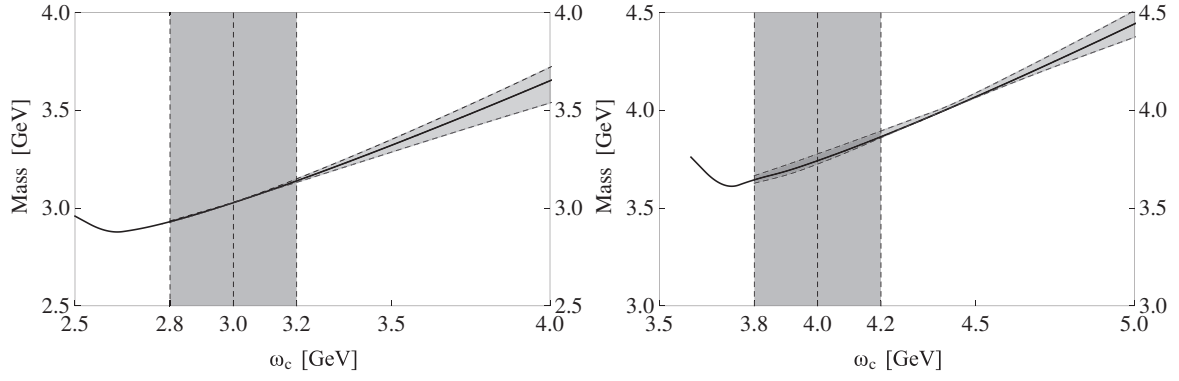


FIG. 10: Variations of $m_{\Lambda_c(3/2^+)}$ (left) and $m_{\Xi_c(3/2^+)}$ (right) with respect to the threshold value ω_c , calculated using the charmed baryon doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$. The shady band is obtained by changing T inside Borel windows. Although the mass curves have minimum against ω_c around 2.6 GeV (left) and 3.7 GeV (right), there exist Borel windows as long as $\omega_c \geq 2.8$ GeV (left) and $\omega_c \geq 3.8$ GeV (right), and the ω_c dependence is still weak and acceptable in the region $2.8 \text{ GeV} < \omega_c < 3.2 \text{ GeV}$ (left) and $3.8 \text{ GeV} < \omega_c < 4.2 \text{ GeV}$ (right).

and similar formulae for the $\Xi_c(5/2^+)$ and $\Xi_c(7/2^+)$, we can obtain

$$\begin{aligned} \frac{1}{14} \left(6m_{\Lambda_c(5/2^+)} + 8m_{\Lambda_c(7/2^+)} \right) &= 3.48^{+0.33}_{-0.18} \text{ GeV} , \\ \frac{1}{14} \left(6m_{\Xi_c(5/2^+)} + 8m_{\Xi_c(7/2^+)} \right) &= 3.80^{+0.20}_{-0.16} \text{ GeV} . \end{aligned} \quad (63)$$

These values are listed in Table I, which are significantly larger than the masses of the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$. We also show their variations with respect to the threshold value ω_c in Fig. 11.

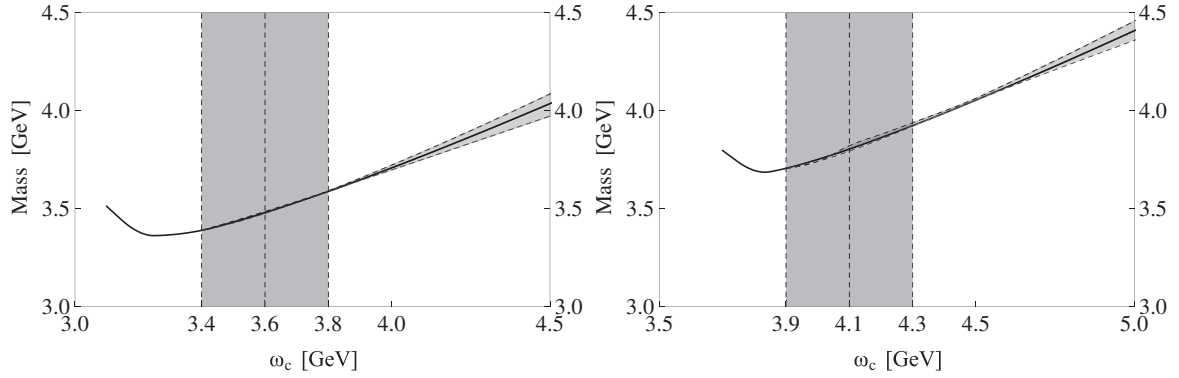


FIG. 11: Variations of $m_{[\Lambda_c, 3, 1, \rho\lambda]}$ (left) and $m_{[\Xi_c, 3, 1, \rho\lambda]}$ (right) with respect to the threshold value ω_c , calculated using the charmed baryon doublet $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$. The shady band is obtained by changing T inside Borel windows. Although the mass curves have minimum against ω_c around 3.2 GeV (left) and 3.8 GeV (right), there exist Borel windows as long as $\omega_c \geq 3.4$ GeV (left) and $\omega_c \geq 3.9$ GeV (right), and the ω_c dependence is still weak and acceptable in the region $3.4 \text{ GeV} < \omega_c < 3.8 \text{ GeV}$ (left) and $3.9 \text{ GeV} < \omega_c < 4.3 \text{ GeV}$ (right). Moreover, these two threshold values satisfy $\omega_c(\Xi_c) - \omega_c(\Lambda_c) = 0.5 \text{ GeV}$.

- The baryon doublet $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ contains $\Lambda_c(3/2^+, 5/2^+)$ and $\Xi_c(3/2^+, 5/2^+)$. We estimate their masses by averaging between $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 1$) and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ ($s_l = 1$ and $j_l = 3$), weighted by the spin-orbital splittings, to be:

$$\begin{aligned} m_{[\Lambda_c, 2, 1, \rho\lambda]} &\sim 3.20 \text{ GeV} , \\ m_{[\Xi_c, 2, 1, \rho\lambda]} &\sim 3.76 \text{ GeV} , \end{aligned} \quad (64)$$

These values are listed in Table I, which are significantly larger than the masses of the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$.

VI. SUMMARY

Summarizing all these results, we have studied the D -wave charmed baryons of $SU(3)$ flavor $\bar{\mathbf{3}}_F$ using the method of QCD sum rules within HQET. We have calculated their masses up to the order $\mathcal{O}(1/m_Q)$ with large theoretical uncertainty, and we have also calculated their mass splittings within the same doublet with much less theoretical uncertainty. Our results suggest that the $\Lambda_c(2880)$, $\Xi_c(3055)$ and $\Xi_c(3080)$ can be well described by the baryon doublet $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$ with $l_\rho = 0$, $l_\lambda = 2$ and $s_l = 0$: a) the $\Lambda_c(2880)$ has $J^P = 5/2^+$, it has a partner state the $\Lambda_c(3/2^+)$ of $J^P = 3/2^+$ with a mass around $2.81^{+0.33}_{-0.18}$ GeV, and their mass difference is 28^{+45}_{-24} MeV; b) the $\Xi_c(3055)$ and $\Xi_c(3080)$ have quantum number $J^P = 3/2^+$ and $5/2^+$, respectively. The first conclusion (a) is consistent with the recent reference [94] by Lü *et al.*

TABLE II: Masses of the D -wave bottom baryons obtained using the baryon doublets $[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$, $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$, $[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$, $[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ and $[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$.

Multiplets	B	ω_c (GeV)	Working region (GeV)	$\bar{\Lambda}$ (GeV)	f (GeV ⁵)	K (GeV ²)	Σ (GeV ²)	Baryons (j^P)	Mass (GeV)	Difference (MeV)
$[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$	Λ_b	2.5	$0.331 < T < 0.381$	1.113	0.012	-2.239	0.014	$\Lambda_b(3/2^+)$	$6.01^{+0.20}_{-0.12}$	6^{+10}_{-5}
								$\Lambda_b(5/2^+)$	$6.01^{+0.20}_{-0.13}$	
	Ξ_b	3.0	$0.341 < T < 0.459$	1.279	0.025	-2.508	0.008	$\Xi_b(3/2^+)$	$6.19^{+0.10}_{-0.12}$	3^{+3}_{-3}
								$\Xi_b(5/2^+)$	$6.19^{+0.10}_{-0.12}$	
$[\bar{\mathbf{3}}_F, 2, 0, \rho\rho]$	Λ_b	3.4	$0.358 < T < 0.499$	1.650	0.065	-1.742	0.011	$\Lambda_b(3/2^+)$	$6.52^{+1.55}_{-0.26}$	5^{+26}_{-4}
								$\Lambda_b(5/2^+)$	$6.52^{+1.58}_{-0.27}$	
	Ξ_b	3.9	$0.502 < T < 0.591$	1.723	0.10	-1.308	0.006	$\Xi_b(3/2^+)$	$6.57^{+0.16}_{-0.12}$	2^{+3}_{-2}
								$\Xi_b(5/2^+)$	$6.57^{+0.16}_{-0.12}$	
$[\bar{\mathbf{3}}_F, 1, 1, \rho\lambda]$	Λ_b	3.0	$0.397 < T < 0.457$	1.335	0.044	-2.116	0.006	$\Lambda_b(1/2^+)$	$6.22^{+0.18}_{-0.12}$	1^{+2}_{-1}
								$\Lambda_b(3/2^+)$	$6.23^{+0.18}_{-0.12}$	
	Ξ_b	4.0	$0.526 < T < 0.599$	1.887	0.21	-2.954	0.003	$\Xi_b(1/2^+)$	$6.82^{+0.11}_{-0.09}$	1^{+1}_{-1}
								$\Xi_b(3/2^+)$	$6.82^{+0.11}_{-0.09}$	
$[\bar{\mathbf{3}}_F, 2, 1, \rho\lambda]$ (estimated)	Λ_b	-	-	-	-	-	-	$\Lambda_b(3/2^+)$	~ 6.36	-
								$\Lambda_b(5/2^+)$		
	Ξ_b	-	-	-	-	-	-	$\Xi_b(3/2^+)$	~ 6.84	-
								$\Xi_b(5/2^+)$		
$[\bar{\mathbf{3}}_F, 3, 1, \rho\lambda]$ (simplified)	Λ_b	3.6	$0.496 < T < 0.542$	1.628	0.022	-2.939	-	$\Lambda_b(5/2^+)$	$6.56^{+0.28}_{-0.15}$	-
								$\Lambda_b(7/2^+)$		
	Ξ_b	4.1	$0.556 < T < 0.609$	1.920	0.045	-3.105	-	$\Xi_b(5/2^+)$	$6.86^{+0.18}_{-0.13}$	-
								$\Xi_b(7/2^+)$		

We have also evaluated the masses of the D bottom baryons of $SU(3)$ flavor $\bar{\mathbf{3}}_F$. The results are listed in Table II, where we have used the pole mass of the bottom quark, i.e., $m_b = 4.78 \pm 0.06$ GeV [1]. We note again that the obtained bottom baryon masses significantly depend on the bottom quark mass, so have large theoretical uncertainty, but their splittings within the same doublet have much less theoretical uncertainty. Especially, the results obtained

by using the baryon doublet $[\bar{\mathbf{3}}_F, 2, 0, \lambda\lambda]$ are

$$\begin{aligned}
m_{\Lambda_b(3/2^+)} &= 6.01_{-0.12}^{+0.20} \text{ GeV}, \\
m_{\Lambda_b(5/2^+)} &= 6.01_{-0.13}^{+0.20} \text{ GeV}, \\
m_{\Lambda_b(5/2^+)} - m_{\Lambda_b(3/2^+)} &= 6_{-5}^{+10} \text{ MeV}, \\
m_{\Xi_b(3/2^+)} &= 6.19_{-0.12}^{+0.10} \text{ GeV}, \\
m_{\Xi_b(5/2^+)} &= 6.19_{-0.12}^{+0.10} \text{ GeV}, \\
m_{\Xi_b(5/2^+)} - m_{\Xi_b(3/2^+)} &= 3_{-3}^{+3} \text{ MeV}.
\end{aligned} \tag{65}$$

We suggest to search for them in further experiments.

To end our paper, we would like to note that not only masses but also decay and production properties are useful to clarify the nature of the heavy baryons, and an experimental project of such studies is planned at J-PARC [95]. Accordingly, in the following studies we plan to study the D -wave charmed baryons of $SU(3)$ flavor $\mathbf{6}_F$ and the D -wave bottom baryons. We also plan to study decay properties of the excited heavy baryons, which can probably provide more useful information.

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Appendix A: Several Projection Operators

For the interpolating field, the projection operator projecting into pure spin 1 is:

$$\Gamma_t^{\mu,\nu} = g_t^{\mu\nu} - \frac{1}{3}\gamma_t^\mu \gamma_t^\nu. \tag{A1}$$

The projection operator projecting into pure spin 2 is:

$$\begin{aligned}
\Gamma_t^{\mu_1\mu_2,\nu_1\nu_2} &= g_t^{\mu_1\nu_1} g_t^{\mu_2\nu_2} + g_t^{\mu_1\nu_2} g_t^{\mu_2\nu_1} - \frac{2}{15} g_t^{\mu_1\mu_2} g_t^{\nu_1\nu_2} \\
&\quad - \frac{1}{3} g_t^{\mu_1\nu_1} \gamma_t^{\mu_2} \gamma_t^{\nu_2} - \frac{1}{3} g_t^{\mu_1\nu_2} \gamma_t^{\mu_2} \gamma_t^{\nu_1} - \frac{1}{3} g_t^{\mu_2\nu_1} \gamma_t^{\mu_1} \gamma_t^{\nu_2} - \frac{1}{3} g_t^{\mu_2\nu_2} \gamma_t^{\mu_1} \gamma_t^{\nu_1} \\
&\quad + \frac{1}{15} \gamma_t^{\mu_1} \gamma_t^{\nu_1} \gamma_t^{\mu_2} \gamma_t^{\nu_2} + \frac{1}{15} \gamma_t^{\mu_1} \gamma_t^{\nu_2} \gamma_t^{\mu_2} \gamma_t^{\nu_1} + \frac{1}{15} \gamma_t^{\mu_2} \gamma_t^{\nu_1} \gamma_t^{\mu_1} \gamma_t^{\nu_2} + \frac{1}{15} \gamma_t^{\mu_2} \gamma_t^{\nu_2} \gamma_t^{\mu_1} \gamma_t^{\nu_1}.
\end{aligned} \tag{A2}$$

The projection operator projecting into pure spin 3 is:

$$\begin{aligned}
\Gamma_t^{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} &= \mathbb{S}'' \left[g_t^{\mu_1\nu_1} g_t^{\mu_2\nu_2} g_t^{\mu_3\nu_3} + c_1 \times g_t^{\mu_1\nu_1} g_t^{\mu_2\mu_3} g_t^{\nu_2\nu_3} + c_2 \times g_t^{\mu_1\nu_1} g_t^{\mu_2\nu_2} \gamma_t^{\mu_3} \gamma_t^{\nu_3} + c_3 \times g_t^{\mu_1\mu_2} g_t^{\nu_1\nu_2} \gamma_t^{\mu_3} \gamma_t^{\nu_3} \right. \\
&\quad \left. + c_4 \times g_t^{\mu_1\nu_1} \gamma_t^{\mu_2} \gamma_t^{\nu_2} \gamma_t^{\mu_3} \gamma_t^{\nu_3} + c_5 \gamma_t^{\mu_1} \gamma_t^{\nu_1} \gamma_t^{\mu_2} \gamma_t^{\nu_2} \gamma_t^{\mu_3} \gamma_t^{\nu_3} \right],
\end{aligned} \tag{A3}$$

where $\mathbb{S}''[\dots]$ denotes symmetrization and subtracting the trace terms in the sets $(\mu_1\mu_2\mu_3)$ and $(\nu_1\nu_2\nu_3)$. The five coefficients $c_{1,2,3,4,5}$ can be obtained by solving $\gamma_t^{\mu_1} \Gamma_t^{\mu_1\mu_2\mu_3,\nu_1\nu_2\nu_3} = 0$, which is not an easy task so we do not solve it here.

In the present study we do not need to always use these projection operators. For example, $J_{3/2,+,\bar{\mathbf{3}}_F,2,0,\rho\rho}^\alpha$ defined in Eq. (2) naturally satisfies

$$\gamma_\alpha^t J_{3/2,+,\bar{\mathbf{3}}_F,2,0,\rho\rho}^\alpha(x) = 0, \tag{A4}$$

so it has pure spin $3/2$.

The situation is much simpler for the two-point correlation function

$$\begin{aligned} \Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2},\beta_1\cdots\beta_{j-1/2}}(\omega) &= i \int d^4x e^{ikx} \langle 0 | T [J_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\alpha_1\cdots\alpha_{j-1/2}}(x) \bar{J}_{j,P,F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}^{\beta_1\cdots\beta_{j-1/2}}(0)] | 0 \rangle \\ &= \mathbb{S}[g_t^{\alpha_1\beta_1} \cdots g_t^{\alpha_{j-1/2}\beta_{j-1/2}}] \frac{1+\not{p}}{2} \Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega) + \cdots, \end{aligned} \quad (\text{A5})$$

that its leading term, $\Pi_{F,j_l,s_l,\rho\rho/\lambda\lambda/\rho\lambda}(\omega)$, only contains the highest spin j component, while \cdots contains other spin components. $\mathbb{S}[\cdots]$ has been defined to denote symmetrization and subtracting the trace terms in the sets $(\alpha_1 \cdots \alpha_{j-1/2})$ and $(\beta_1 \cdots \beta_{j-1/2})$.

Appendix B: Other Sum Rules

In this appendix we show the sum rules for other currents with different quark contents:

$$\Pi_{\Lambda_c,2,0,\rho\rho} = f_{\Lambda_c,2,0,\rho\rho}^2 e^{-2\bar{\Lambda}_{\Lambda_c,2,0,\rho\rho}/T} = \int_0^{\omega_c} \left[\frac{5}{145152\pi^4} \omega^9 - \frac{5\langle g_s^2 GG \rangle}{1728\pi^4} \omega^5 \right] e^{-\omega/T} d\omega, \quad (\text{B1})$$

$$f_{\Lambda_c,2,0,\rho\rho}^2 K_{\Lambda_c,2,0,\rho\rho} e^{-2\bar{\Lambda}_{\Lambda_c,2,0,\rho\rho}/T} = \int_0^{\omega_c} \left[-\frac{41}{6386688\pi^4} \omega^{11} + \frac{59\langle g_s^2 GG \rangle}{90720\pi^4} \omega^7 \right] e^{-\omega/T} d\omega, \quad (\text{B2})$$

$$f_{\Lambda_c,2,0,\rho\rho}^2 \Sigma_{\Lambda_c,2,0,\rho\rho} e^{-2\bar{\Lambda}_{\Lambda_c,2,0,\rho\rho}/T} = \int_0^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{24192\pi^4} \omega^7 \right] e^{-\omega/T} d\omega. \quad (\text{B3})$$

$$\Pi_{\Xi_c,2,0,\rho\rho} = f_{\Xi_c,2,0,\rho\rho}^2 e^{-2\bar{\Lambda}_{\Xi_c,2,0,\rho\rho}/T} \quad (\text{B4})$$

$$\begin{aligned} &= \int_{2m_s}^{\omega_c} \left[\frac{5}{145152\pi^4} \omega^9 - \frac{m_s^2}{672\pi^4} \omega^7 - \frac{m_s \langle \bar{q}q \rangle}{72\pi^2} \omega^5 + \frac{m_s \langle \bar{s}s \rangle}{48\pi^2} \omega^5 \right. \\ &\quad \left. - \frac{5\langle g_s^2 GG \rangle}{1728\pi^4} \omega^5 + \frac{5m_s^2 \langle g_s^2 GG \rangle}{192\pi^4} \omega^3 - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{72\pi^2} \omega \right] e^{-\omega/T} d\omega, \end{aligned}$$

$$f_{\Xi_c,2,0,\rho\rho}^2 K_{\Xi_c,2,0,\rho\rho} e^{-2\bar{\Lambda}_{\Xi_c,2,0,\rho\rho}/T} \quad (\text{B5})$$

$$\begin{aligned} &= \int_{2m_s}^{\omega_c} \left[-\frac{41}{6386688\pi^4} \omega^{11} + \frac{197m_s^2}{483840\pi^4} \omega^9 + \frac{37m_s \langle \bar{q}q \rangle}{5040\pi^2} \omega^7 - \frac{277m_s \langle \bar{s}s \rangle}{20160\pi^2} \omega^7 + \frac{11m_s \langle g_s \bar{q}\sigma Gq \rangle}{180\pi^2} \omega^5 \right. \\ &\quad \left. + \frac{1921\langle g_s^2 GG \rangle}{2903040\pi^4} \omega^7 - \frac{7169m_s^2 \langle g_s^2 GG \rangle}{552960\pi^4} \omega^5 - \frac{13m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{216\pi^2} \omega^3 + \frac{2381m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{20736\pi^2} \omega^3 \right. \\ &\quad \left. - \frac{121m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{1728\pi^2} \omega \right] e^{-\omega/T} d\omega, \end{aligned}$$

$$f_{\Xi_c,2,0,\rho\rho}^2 \Sigma_{\Xi_c,2,0,\rho\rho} e^{-2\bar{\Lambda}_{\Xi_c,2,0,\rho\rho}/T} \quad (\text{B6})$$

$$= \int_{2m_s}^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{24192\pi^4} \omega^7 - \frac{m_s^2 \langle g_s^2 GG \rangle}{1536\pi^4} \omega^5 + \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{864\pi^2} \omega^3 \right] e^{-\omega/T} d\omega.$$

$$\Pi_{\Lambda_c,1,1,\rho\lambda} = f_{\Lambda_c,1,1,\rho\lambda}^2 e^{-2\bar{\Lambda}_{\Lambda_c,1,1,\rho\lambda}/T} = \int_0^{\omega_c} \left[\frac{13}{161280\pi^4} \omega^9 - \frac{43\langle g_s^2 GG \rangle}{15360\pi^4} \omega^5 \right] e^{-\omega/T} d\omega, \quad (\text{B7})$$

$$f_{\Lambda_c,1,1,\rho\lambda}^2 K_{\Lambda_c,1,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Lambda_c,1,1,\rho\lambda}/T} = \int_0^{\omega_c} \left[-\frac{461}{17740800\pi^4} \omega^{11} + \frac{383\langle g_s^2 GG \rangle}{322560\pi^4} \omega^7 \right] e^{-\omega/T} d\omega, \quad (\text{B8})$$

$$f_{\Lambda_c,1,1,\rho\lambda}^2 \Sigma_{\Lambda_c,1,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Lambda_c,1,1,\rho\lambda}/T} = \int_0^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{26880\pi^4} \omega^7 \right] e^{-\omega/T} d\omega. \quad (\text{B9})$$

$$\Pi_{\Xi_c,1,1,\rho\lambda} = f_{\Xi_c,1,1,\rho\lambda}^2 e^{-2\bar{\Lambda}_{\Xi_c,1,1,\rho\lambda}/T} \quad (\text{B10})$$

$$= \int_{2m_s}^{\omega_c} \left[\frac{13}{161280\pi^4} \omega^9 - \frac{9m_s^2}{2240\pi^4} \omega^7 - \frac{m_s \langle \bar{q}q \rangle}{16\pi^2} \omega^5 + \frac{3m_s \langle \bar{s}s \rangle}{32\pi^2} \omega^5 - \frac{3m_s \langle g_s \bar{q}\sigma Gq \rangle}{8\pi^2} \omega^3 \right. \\ \left. - \frac{43 \langle g_s^2 GG \rangle}{15360\pi^4} \omega^5 + \frac{9m_s^2 \langle g_s^2 GG \rangle}{256\pi^4} \omega^3 - \frac{9m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{64\pi^2} \omega \right] e^{-\omega/T} d\omega, \quad (\text{B11})$$

$$f_{\Xi_c,1,1,\rho\lambda}^2 K_{\Xi_c,1,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Xi_c,1,1,\rho\lambda}/T} \\ = \int_{2m_s}^{\omega_c} \left[-\frac{461}{17740800\pi^4} \omega^{11} + \frac{283m_s^2}{161280\pi^4} \omega^9 + \frac{m_s \langle \bar{q}q \rangle}{32\pi^2} \omega^7 - \frac{29m_s \langle \bar{s}s \rangle}{448\pi^2} \omega^7 + \frac{5m_s \langle g_s \bar{q}\sigma Gq \rangle}{16\pi^2} \omega^5 \right. \\ \left. + \frac{383 \langle g_s^2 GG \rangle}{322560\pi^4} \omega^7 - \frac{191m_s^2 \langle g_s^2 GG \rangle}{7680\pi^4} \omega^5 - \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{q}q \rangle}{72\pi^2} \omega^3 + \frac{133m_s \langle \bar{s}s \rangle \langle g_s^2 GG \rangle}{768\pi^2} \omega^3 \right. \\ \left. - \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle \langle g_s^2 GG \rangle}{48\pi^2} \omega \right] e^{-\omega/T} d\omega, \quad (\text{B12})$$

$$f_{\Xi_c,1,1,\rho\lambda}^2 \Sigma_{\Xi_c,1,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Xi_c,1,1,\rho\lambda}/T} \\ = \int_{2m_s}^{\omega_c} \left[\frac{\langle g_s^2 GG \rangle}{26880\pi^4} \omega^7 - \frac{m_s^2 \langle g_s^2 GG \rangle}{960\pi^4} \omega^5 + \frac{5m_s \langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{288\pi^2} \omega^3 \right] e^{-\omega/T} d\omega. \quad (\text{B13})$$

$$\Pi_{\Lambda_c,3,1,\rho\lambda} = f_{\Lambda_c,3,1,\rho\lambda}^2 e^{-2\bar{\Lambda}_{\Lambda_c,3,1,\rho\lambda}/T} = \int_0^{\omega_c} \left[\frac{1}{322560\pi^4} \omega^9 - \frac{\langle g_s^2 GG \rangle}{3840\pi^4} \omega^5 \right] e^{-\omega/T} d\omega, \quad (\text{B14})$$

$$f_{\Lambda_c,3,1,\rho\lambda}^2 K_{\Lambda_c,3,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Lambda_c,3,1,\rho\lambda}/T} = \int_0^{\omega_c} \left[-\frac{1}{1075200\pi^4} \omega^{11} + \frac{53 \langle g_s^2 GG \rangle}{552960\pi^4} \omega^7 \right] e^{-\omega/T} d\omega. \quad (\text{B15})$$

$$\Pi_{\Xi_c,3,1,\rho\lambda} = f_{\Xi_c,3,1,\rho\lambda}^2 e^{-2\bar{\Lambda}_{\Xi_c,3,1,\rho\lambda}/T} \\ = \int_{2m_s}^{\omega_c} \left[\frac{1}{322560\pi^4} \omega^9 - \frac{m_s^2}{6720\pi^4} \omega^7 - \frac{m_s \langle \bar{q}q \rangle}{480\pi^2} \omega^5 + \frac{m_s \langle \bar{s}s \rangle}{320\pi^2} \omega^5 - \frac{m_s \langle g_s \bar{q}\sigma Gq \rangle}{96\pi^2} \omega^3 \right. \\ \left. - \frac{\langle g_s^2 GG \rangle}{3840\pi^4} \omega^5 + \frac{m_s^2 \langle g_s^2 GG \rangle}{512\pi^4} \omega^3 - \frac{m_s \langle \bar{s}s \rangle \langle g_s^2 GG \rangle}{128\pi^2} \omega \right] e^{-\omega/T} d\omega, \quad (\text{B16})$$

$$f_{\Xi_c,3,1,\rho\lambda}^2 K_{\Xi_c,3,1,\rho\lambda} e^{-2\bar{\Lambda}_{\Xi_c,3,1,\rho\lambda}/T} \\ = \int_{2m_s}^{\omega_c} \left[-\frac{1}{1075200\pi^4} \omega^{11} + \frac{29m_s^2}{483840\pi^4} \omega^9 + \frac{m_s \langle \bar{q}q \rangle}{960\pi^2} \omega^7 - \frac{9m_s \langle \bar{s}s \rangle}{4480\pi^2} \omega^7 + \frac{3m_s \langle g_s \bar{q}\sigma Gq \rangle}{320\pi^2} \omega^5 \right. \\ \left. + \frac{53 \langle g_s^2 GG \rangle}{552960\pi^4} \omega^7 - \frac{217m_s^2 \langle g_s^2 GG \rangle}{184320\pi^4} \omega^5 - \frac{5m_s \langle \bar{q}q \rangle \langle g_s^2 GG \rangle}{1728\pi^2} \omega^3 + \frac{65m_s \langle \bar{s}s \rangle \langle g_s^2 GG \rangle}{6912\pi^2} \omega^3 \right. \\ \left. - \frac{m_s \langle g_s^2 GG \rangle \langle g_s \bar{q}\sigma Gq \rangle}{1152\pi^2} \omega \right] e^{-\omega/T} d\omega.$$

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- [1] K. A. Olive *et al.* [Particle Data Group], Review of Particle Physics, Chin. Phys. C **38**, 090001 (2014).
[2] H. Albrecht *et al.* [ARGUS Collaboration], Observation of a new charmed baryon, Phys. Lett. B **317**, 227 (1993).
[3] P. L. Frabetti *et al.* [E687 Collaboration], An Observation of an excited state of the Λ_c^+ baryon, Phys. Rev. Lett. **72**, 961 (1994).
[4] K. W. Edwards *et al.* [CLEO Collaboration], Observation of excited baryon states decaying to $\Lambda_c^+ \pi^+ \pi^-$, Phys. Rev. Lett. **74**, 3331 (1995).
[5] J. P. Alexander *et al.* [CLEO Collaboration], Evidence of new states decaying into $\Xi_c^* \pi$, Phys. Rev. Lett. **83**, 3390 (1999).
[6] M. Artuso *et al.* [CLEO Collaboration], Observation of new states decaying into $\Lambda_c^+ \pi^- \pi^+$, Phys. Rev. Lett. **86**, 4479 (2001).
[7] B. Aubert *et al.* [BaBar Collaboration], Observation of a charmed baryon decaying to $D^0 p$ at a mass near 2.94-GeV/c², Phys. Rev. Lett. **98**, 012001 (2007).
[8] K. Abe *et al.* [Belle Collaboration], Experimental constraints on the possible J^P quantum numbers of the $\Lambda_c(2880)^+$, Phys. Rev. Lett. **98**, 262001 (2007).

- [9] R. Mizuk *et al.* [Belle Collaboration], Observation of an isotriplet of excited charmed baryons decaying to $\Lambda_c^+ \pi$, Phys. Rev. Lett. **94**, 122002 (2005).
- [10] B. Aubert *et al.* [BaBar Collaboration], A Study of $\bar{B} \rightarrow \Xi_c \bar{\Lambda}_c^-$ and $\bar{B} \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- \bar{K}$ decays at BABAR, Phys. Rev. D **77**, 031101 (2008).
- [11] R. Chistov *et al.* [Belle Collaboration], Observation of new states decaying into $\Lambda_c^+ K^- \pi^+$ and $\Lambda_c^+ K_S^0 \pi^-$, Phys. Rev. Lett. **97**, 162001 (2006).
- [12] J. Yelton *et al.* [Belle Collaboration], Study of Excited Ξ_c States Decaying into Ξ_c^0 and Ξ_c^+ Baryons, Phys. Rev. D **94**, 052011 (2016).
- [13] B. Aubert *et al.* [BaBar Collaboration], A Study of Excited Charm-Strange Baryons with Evidence for new Baryons $\Xi_c(3055)^+$ and $\Xi_c(3123)^+$, Phys. Rev. D **77**, 012002 (2008).
- [14] Y. Kato *et al.* [Belle Collaboration], Studies of charmed strange baryons in the ΛD final state at Belle, Phys. Rev. D **94**, 032002 (2016).
- [15] D. Ebert, R. N. Faustov and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, Phys. Rev. D **84**, 014025 (2011).
- [16] B. Chen, K. W. Wei and A. Zhang, Assignments of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture, Eur. Phys. J. A **51**, 82 (2015).
- [17] H. Y. Cheng, Charmed baryons circa 2015, Front. Phys. (Beijing) **10**, no. 6, 101406 (2015).
- [18] H. Y. Cheng and C. K. Chua, Strong Decays of Charmed Baryons in Heavy Hadron Chiral Perturbation Theory, Phys. Rev. D **75**, 014006 (2007).
- [19] H. Garcilazo, J. Vijande and A. Valcarce, Faddeev study of heavy baryon spectroscopy, J. Phys. G **34**, 961 (2007).
- [20] S. M. Gerasyuta and E. E. Matskevich, Charmed $(70, 1^-)$ baryon multiplet, Int. J. Mod. Phys. E **17**, 585 (2008).
- [21] X. H. Zhong and Q. Zhao, Charmed baryon strong decays in a chiral quark model, Phys. Rev. D **77**, 074008 (2008).
- [22] A. Selem and F. Wilczek, Hadron systematics and emergent diquarks, hep-ph/0602128.
- [23] H. X. Chen, W. Chen, X. Liu, Y. R. Liu and S. L. Zhu, arXiv:1609.08928 [hep-ph].
- [24] S. Capstick and N. Isgur, Baryons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D **34**, 2809 (1986).
- [25] D. Ebert, R. N. Faustov and V. O. Galkin, Masses of excited heavy baryons in the relativistic quark model, Phys. Lett. B **659**, 612 (2008).
- [26] P. G. Ortega, D. R. Entem and F. Fernandez, Quark model description of the $\Lambda_c(2940)^+$ as a molecular $D^* N$ state and the possible existence of the $\Lambda_b(6248)$, Phys. Lett. B **718**, 1381 (2013).
- [27] Z. Shah, K. Thakkar, A. K. Rai and P. C. Vinodkumar, arXiv:1609.08464 [nucl-th].
- [28] K. Thakkar, Z. Shah, A. K. Rai and P. C. Vinodkumar, arXiv:1610.00411 [nucl-th].
- [29] E. E. Jenkins, Heavy baryon masses in the $1/m_Q$ and $1/N_c$ expansions, Phys. Rev. D **54**, 4515 (1996).
- [30] L. A. Copley, N. Isgur and G. Karl, Charmed Baryons in a Quark Model with Hyperfine Interactions, Phys. Rev. D **20**, 768 (1979) [Erratum-ibid. D **23**, 817 (1981)].
- [31] M. Karliner, B. Keren-Zur, H. J. Lipkin and J. L. Rosner, The Quark Model and b Baryons, Annals Phys. **324**, 2 (2009).
- [32] R. Roncaglia, D. B. Lichtenberg and E. Predazzi, Predicting the masses of baryons containing one or two heavy quarks, Phys. Rev. D **52**, 1722 (1995).
- [33] W. Roberts and M. Pervin, Heavy baryons in a quark model, Int. J. Mod. Phys. A **23**, 2817 (2008).
- [34] C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo and L. Tolos, Odd parity bottom-flavored baryon resonances, Phys. Rev. D **87**, 034032 (2013).
- [35] W. H. Liang, C. W. Xiao and E. Oset, Baryon states with open beauty in the extended local hidden gauge approach, Phys. Rev. D **89**, 054023 (2014).
- [36] J. X. Lu, Y. Zhou, H. X. Chen, J. J. Xie and L. S. Geng, Dynamically generated $J^P = 1/2^-(3/2^-)$ singly charmed and bottom heavy baryons, Phys. Rev. D **92**, 014036 (2015).
- [37] K. C. Bowler *et al.* [UKQCD Collaboration], Heavy baryon spectroscopy from the lattice, Phys. Rev. D **54**, 3619 (1996).
- [38] T. Burch, C. Hagen, C. B. Lang, M. Limmer and A. Schäfer, Excitations of single-beauty hadrons, Phys. Rev. D **79**, 014504 (2009).
- [39] Z. S. Brown, W. Detmold, S. Meinel and K. Orginos, Charmed bottom baryon spectroscopy from lattice QCD, Phys. Rev. D **90**, no. 9, 094507 (2014).
- [40] H. Nagahiro, S. Yasui, A. Hosaka, M. Oka and H. Noumi, arXiv:1609.01085 [hep-ph].
- [41] S. H. Kim, A. Hosaka, H. C. Kim, H. Noumi and K. Shirotori, Pion induced Reactions for Charmed Baryons, PTEP **2014**, 103D01 (2014).
- [42] J. G. Körner, M. Kramer and D. Pirjol, Heavy baryons, Prog. Part. Nucl. Phys. **33**, 787 (1994).
- [43] S. Bianco, F. L. Fabbri, D. Benson and I. Bigi, A Cicerone for the physics of charm, Riv. Nuovo Cim. **26N7**, 1 (2003).
- [44] E. Klempt and J. M. Richard, Baryon spectroscopy, Rev. Mod. Phys. **82**, 1095 (2010).
- [45] V. Crede and W. Roberts, Progress towards understanding baryon resonances, Rept. Prog. Phys. **76**, 076301 (2013).
- [46] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka and S. L. Zhu, Bottom baryons, Phys. Rev. D **77**, 014031 (2008).
- [47] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu and S. L. Zhu, P-wave charmed baryons from QCD sum rules, Phys. Rev. D **91**, 054034 (2015).
- [48] Q. Mao, H. X. Chen, W. Chen, A. Hosaka, X. Liu and S. L. Zhu, QCD sum rule calculation for P-wave bottom baryons, Phys. Rev. D **92**, 114007 (2015).
- [49] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, QCD And Resonance Physics. Sum Rules, Nucl. Phys. B **147**, 385 (1979).
- [50] L. J. Reinders, H. Rubinstein and S. Yazaki, Hadron Properties From QCD Sum Rules, Phys. Rept. **127**, 1 (1985).

- [51] B. Grinstein, The Static Quark Effective Theory, Nucl. Phys. B **339**, 253 (1990).
- [52] E. Eichten and B. R. Hill, An Effective Field Theory for the Calculation of Matrix Elements Involving Heavy Quarks, Phys. Lett. B **234**, 511 (1990).
- [53] A. F. Falk, H. Georgi, B. Grinstein and M. B. Wise, Heavy Meson Form-factors From QCD, Nucl. Phys. B **343**, 1 (1990).
- [54] E. Bagan, P. Ball, V. M. Braun and H. G. Dosch, QCD sum rules in the effective heavy quark theory, Phys. Lett. B **278**, 457 (1992).
- [55] M. Neubert, Heavy meson form-factors from QCD sum rules, Phys. Rev. D **45**, 2451 (1992).
- [56] M. Neubert, Heavy quark symmetry, Phys. Rept. **245**, 259 (1994).
- [57] D. J. Broadhurst and A. G. Grozin, Operator product expansion in static quark effective field theory: Large perturbative correction, Phys. Lett. B **274**, 421 (1992).
- [58] P. Ball and V. M. Braun, Next-to-leading order corrections to meson masses in the heavy quark effective theory, Phys. Rev. D **49**, 2472 (1994).
- [59] T. Huang and C. W. Luo, Light quark dependence of the Isgur-Wise function from QCD sum rules, Phys. Rev. D **50**, 5775 (1994).
- [60] Y. B. Dai, C. S. Huang, M. Q. Huang and C. Liu, QCD sum rules for masses of excited heavy mesons, Phys. Lett. B **390**, 350 (1997).
- [61] Y. B. Dai, C. S. Huang and H. Y. Jin, Bethe-Salpeter wave functions and transition amplitudes for heavy mesons, Z. Phys. C **60**, 527 (1993).
- [62] Y. B. Dai, C. S. Huang and M. Q. Huang, $O(1/m_Q)$ order corrections to masses of excited heavy mesons from QCD sum rules, Phys. Rev. D **55**, 5719 (1997).
- [63] P. Colangelo, F. De Fazio and N. Paver, Universal $\tau_{1/2}(y)$ Isgur-Wise function at the next-to-leading order in QCD sum rules, Phys. Rev. D **58**, 116005 (1998).
- [64] Y. B. Dai, C. S. Huang, C. Liu and S. L. Zhu, Understanding the $D_{sJ}^+(2317)$ and $D_{sJ}^+(2460)$ with sum rules in HQET, Phys. Rev. D **68**, 114011 (2003).
- [65] D. Zhou, E. L. Cui, H. X. Chen, L. S. Geng, X. Liu and S. L. Zhu, The D-wave heavy-light mesons from QCD sum rules, Phys. Rev. D **90**, 114035 (2014).
- [66] D. Zhou, H. X. Chen, L. S. Geng, X. Liu and S. L. Zhu, F-wave heavy-light meson spectroscopy in QCD sum rules and heavy quark effective theory, Phys. Rev. D **92**, 114015 (2015).
- [67] E. V. Shuryak, Hadrons Containing a Heavy Quark and QCD Sum Rules, Nucl. Phys. B **198**, 83 (1982).
- [68] A. G. Grozin and O. I. Yakovlev, Baryonic currents and their correlators in the heavy quark effective theory, Phys. Lett. B **285**, 254 (1992).
- [69] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, Baryon sum rules in the heavy quark effective theory, Phys. Lett. B **301**, 243 (1993).
- [70] Y. B. Dai, C. S. Huang, C. Liu and C. D. Lu, $1/m$ corrections to heavy baryon masses in the heavy quark effective theory sum rules, Phys. Lett. B **371**, 99 (1996).
- [71] Y. B. Dai, C. S. Huang, M. Q. Huang and C. Liu, QCD sum rule analysis for the $\Lambda_b \rightarrow \Lambda_c$ semileptonic decay, Phys. Lett. B **387**, 379 (1996).
- [72] S. Groote, J. G. Körner and O. I. Yakovlev, QCD sum rules for heavy baryons at next-to-leading order in α_s , Phys. Rev. D **55**, 3016 (1997).
- [73] S. L. Zhu, Strong and electromagnetic decays of p wave heavy baryons Λ_{c1} , Λ_{c1}^* , Phys. Rev. D **61**, 114019 (2000).
- [74] J. P. Lee, C. Liu and H. S. Song, QCD sum rule analysis of excited Λ_c mass parameter, Phys. Lett. B **476**, 303 (2000).
- [75] C. S. Huang, A. I. Zhang and S. L. Zhu, Excited heavy baryon masses in HQET QCD sum rules, Phys. Lett. B **492**, 288 (2000).
- [76] D. W. Wang and M. Q. Huang, Excited heavy baryon masses to order Λ_{QCD}/m_Q from QCD sum rules, Phys. Rev. D **68**, 034019 (2003).
- [77] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, Spectra of heavy baryons from QCD spectral sum rules, Phys. Lett. B **287**, 176 (1992).
- [78] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, The Heavy baryons Σ_c Σ_b from QCD spectral sum rules, Phys. Lett. B **278**, 367 (1992).
- [79] F. O. Duraes and M. Nielsen, QCD sum rules study of Ξ_c and Ξ_b baryons, Phys. Lett. B **658**, 40 (2007).
- [80] Z. G. Wang, Analysis of $\Omega_c^*(css)$ and $\Omega_b^*(bss)$ with QCD sum rules, Eur. Phys. J. C **54**, 231 (2008).
- [81] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rept. **639**, 1 (2016).
- [82] H. X. Chen, W. Chen, X. Liu, T. G. Steele and S. L. Zhu, Towards exotic hidden-charm pentaquarks in QCD, Phys. Rev. Lett. **115**, 172001 (2015).
- [83] C. Chen, X. L. Chen, X. Liu, W. Z. Deng and S. L. Zhu, Strong decays of charmed baryons, Phys. Rev. D **75**, 094017 (2007).
- [84] R. Mertig, M. Böhm and A. Denner, FEYN CALC: Computer algebraic calculation of Feynman amplitudes, Comput. Phys. Commun. **64**, 345 (1991).
- [85] K. C. Yang, W. Y. P. Hwang, E. M. Henley and L. S. Kisslinger, QCD sum rules and neutron proton mass difference, Phys. Rev. D **47**, 3001 (1993).
- [86] W. Y. P. Hwang and K. C. Yang, Phys. Rev. D **49**, 460 (1994).
- [87] S. Narison, QCD as a theory of hadrons (from partons to confinement), Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **17**, 1 (2002).

- [88] V. Gimenez, V. Lubicz, F. Mescia, V. Porretti and J. Reyes, Operator product expansion and quark condensate from lattice QCD in coordinate space, *Eur. Phys. J. C* **41**, 535 (2005).
- [89] M. Jamin, Flavour-symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann-Oakes-Renner relation, *Phys. Lett. B* **538**, 71 (2002).
- [90] B. L. Ioffe and K. N. Zyablyuk, Gluon condensate in charmonium sum rules with 3-loop corrections, *Eur. Phys. J. C* **27**, 229 (2003).
- [91] A. A. Ovchinnikov and A. A. Pivovarov, QCD Sum Rule Calculation Of The Quark Gluon Condensate, *Sov. J. Nucl. Phys.* **48**, 721 (1988) [*Yad. Fiz.* **48**, 1135 (1988)].
- [92] P. Colangelo and A. Khodjamirian, “*At the Frontier of Particle Physics/Handbook of QCD*” (World Scientific, Singapore, 2001), Volume 3, 1495.
- [93] H. X. Chen, E. L. Cui, W. Chen, T. G. Steele and S. L. Zhu, QCD sum rule study of the $d^*(2380)$, *Phys. Rev. C* **91**, 025204 (2015).
- [94] Q. F. Lü, Y. Dong, X. Liu and T. Matsuki, Puzzle of the Λ_c spectrum, arXiv:1610.09605 [hep-ph].
- [95] Charmed Baryon Spectroscopy via the (π^-, D^{*-}) reaction (2012). (Available at: http://www.j-parc.jp/researcher/Hadron/en/Proposal_e.html#1301). J-PARC P50 proposal.